Entry into Innovation Areas *

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Abstract

This paper investigates the multi-area entry decisions of publicly listed firms in the United States electronics industry from 1990 to 2019. I apply Latent Dirichlet Allocation to over 630,000 patent titles and abstracts to endogenously define twenty innovation areas and merge these with market-implied patent valuations. The data reveal three empirical patterns. First, innovation occurs across multiple domains simultaneously. Second, the number of active competitors varies across areas. Third, the industry focus shifted from physical hardware to connectivity technologies. To explain these patterns, I estimate a static simultaneous-move game of firm entry using a partial identification approach with moment inequalities, which accommodates multiple equilibria and the combinatorial choice set. The 99 percent confidence sets support four conclusions. First, firms enter areas where expected patent valuations are higher. Second, holding those valuations fixed, additional rivals reduce expected profit. Third, firms also benefit from occupying more areas at once. Fourth, a large per-area entry cost, together with rival congestion, still limits how many areas a firm enters. Starting from observed portfolios, between 13 and 16 percent of firm-years already have at least one extra area that looks worth occupying at the estimated costs. A 25 percent reduction in the per-area cost raises that share to between 22 and 27 percent, and a 50 percent reduction raises it to between 33 and 41 percent. A 50 percent reduction in how costly rivals are raises that share to between 15 and 21 percent. In a case study of targeted cost reductions, entry rises in semiconductor areas such as "Semiconductor Memory Architecture" and "Semiconductor Lithography and Fabrication Processes." The offset is lower entry in other fields, including "Wireless Base-Station and Mobile Communications."

Keywords: Entry; Innovation; Patents; Technological diversification. **JEL:** L25, O31, O34

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1 Introduction

Firms operating in the modern technology sector face a complex strategic problem: deciding which specific innovation and technological areas to enter and invest their research and development resources. This paper develops an empirical model to untangle the economic determinants that govern these multi-area entry decisions. Using a comprehensive panel of publicly listed firms in the United States electronics industry from 1990 to 2019, I quantify the roles of expected innovation valuation, congestion effects, the firm's internal innovation portfolio, and the entry cost in shaping the entry decision. By estimating the model, this paper seeks to explain the observed multi-area bundling nature of research and the limits of the firm's technological expansion.

To analyze these dynamics, I construct a novel dataset that merges financial information from publicly listed electronics firms with rich, unstructured patent texts. Standard administrative patent classifications often fail to capture the evolving nature of technology markets. To address this limitation, I deploy an unsupervised machine learning algorithm, Latent Dirichlet Allocation (LDA), on the text of over 630,000 patent titles and abstracts. Coupled with iterative grid search optimization, this approach allows the data to endogenously define twenty distinct innovation areas based on semantic word co-occurrences. I then integrate market-implied patent valuations from Kogan et al. (2017), which are derived from stock market event studies, to assign an expected monetary value to these technological domains.

The use of topic modeling is a deliberate departure from rigid administrative taxonomies. Traditional classification systems rely on manual, subjective assignments that struggle to accommodate rapid structural shifts in industries like electronics. More critically for structural estimation, these administrative systems impose discrete, categorical boundaries without providing a mathematical weighting scheme for inventions that span multiple fields. By utilizing a generative probabilistic model on the patent texts themselves, I obtain a continuous distribution that maps a single patent across multiple thematic domains. This provides an empirically tractable, data-driven definition of the market choice set that a firm faces.

Using the innovation areas derived from LDA, I document several stylized facts regarding the dynamics of the U.S. electronics sector. First, multi-domain bundling is a dominant feature of the industry. Innovation rarely occurs in isolated silos; the average patent spans 3.58 technological domains, and the median firm-year occupies five innovation areas, competing in several distinct markets simultaneously. These empirical patterns suggest that entry is a complex portfolio optimization problem subject to economies or diseconomies of scope.

Second, the number of active competitors is heterogeneous across innovation areas. Foundational domains such as "Integrated Circuit Design, Verification, and Testing" and "Audio and Speech-Signal Processing" are consistently occupied by many players. In contrast, specific hardware domains like "Display Panel Tech (LCD/OLED)" maintain a sparse competitive environment.

Third, the focus of the industry has rotated over time. The composition of active firms reveals a structural shift across the sample period. Throughout the timeline, the center of competitive activity shifted away from heavy physical innovations and materials engineering, marked by a declining share of firms operating in ”Thin-Film Materials, Deposition and Chemistry” and ”Rotational Electromechanics and Magnetic Actuation”. Concurrently, the industry moved toward modern connectivity and consumer technologies, with a rising share of firms in ”Wireless Base-Station and Mobile Communications” and ”Networking Protocols and Data Infrastructure.” Patent value moved with that rotation, toward ”Mobile UI and Content Services,” ”Wireless Base-Station and Mobile Communications,” and ”Power-Supply and Power-Management Electronics.”

To rationalize these stylized facts, I build a static, complete-information, simultaneous-move game of firm entry. In this model, a firm chooses a bundle of innovation areas to maximize its total profit. The profit derived from entering any single area depends on four components: the expected valuation of the patents within that area, a congestion effect driven by the number of active rival firms, a per-area fixed entry cost, and an area-specific shock. Because a firm typically enters multiple domains simultaneously, its total profit is also subject to an organizational portfolio term. This portfolio term depends on the total count of entered areas, capturing either the synergistic economies of scope or the managerial coordination costs of maintaining a broad technological footprint.

Estimating the parameters of this profit function presents several econometric challenges. First, because firms choose a bundle of areas, the strategy space explodes combinatorially. Second, strategic entry games generate multiple equilibria, which forces standard point identification methods to rely on arbitrary equilibrium selection rules. Finally, point identifying models with expected valuations requires the researcher to exactly specify the firm’s unobserved internal information set, risking severe model misspecification. To overcome these compounding hurdles, I abandon point identification and adopt a partial identification approach utilizing moment inequalities.

I construct these inequalities from the necessary conditions for a Nash equilibrium, using revealed preference. Firms maximize expected profit given the publicly observed state. Area-specific shocks realize after entry, so the econometrician does not need to specify private types or an equilibrium-selection rule. Specifically, I evaluate the conditions where a firm unilaterally alters its observed portfolio by adding or dropping a subset of innovation areas, or by swapping one entered area for a non-entered area[1]. By analyzing these specific deviations, the estimation isolates the marginal incentives driving entry. Finally, to manage measurement error in the expected value of innovation, I execute a first-stage projection that recovers the firm’s smoothed ex ante expectations prior to evaluating the moment bounds.

The 99 percent confidence sets discipline four claims about entry. First, firms enter areas where

[1] While the theoretical framework permits deviations of any size, I empirically restrict the add and drop deviations to pairs and triplets of areas. A one-area add and a one-area drop shift the portfolio term by amounts that are nearly collinear with the per-area cost, given observed portfolio size, and the identified set then collapses. Pair and triplet deviations break that collinearity and remain frequent enough to retain power.

expected patent valuations are higher. Second, holding those valuations fixed, additional rivals reduce expected profit. The sets exclude any specification in which extra rivals raise expected profit, or leave it unchanged. The implied congestion penalty is steep when few firms occupy an area and flattens once the area is already crowded. A reduced-form regression of entry on rival counts is also positive, including in a specification that already includes expected valuation.

Third, firms also gain from occupying more areas at once. The sets exclude a zero or negative scope benefit: the value of adding another area rises with how many areas the firm already holds. A reduced-form regression of entry on portfolio size is likewise positive. The inequalities isolate that relationship from unobserved firm capability. Under pairwise interactions among entered areas, the 99 percent set on the scope term is nonempty and strictly positive. Treating extra areas as diminishing synergies leaves that set empty when all three instruments and the largest subset sample are used. Fourth, firms still leave many areas unentered because the per-area entry cost is large. Rival congestion and that fixed cost limit technological expansion.

Starting from the observed entry pattern, between 13 and 16 percent of firm-years already have at least one extra area that looks worth occupying at the estimated costs. A 25 percent reduction in the per-area cost raises that share to between 22 and 27 percent, and a 50 percent reduction raises it to between 33 and 41 percent. A 50 percent reduction in how costly rivals are raises that share to between 15 and 21 percent. In a case study of targeted cost reductions, entry rises in "Semiconductor Memory Architecture" and "Semiconductor Lithography and Fabrication Processes." The offset is lower entry in other fields, including "Wireless Base-Station and Mobile Communications."

***Related Literature*.**—This paper contributes to several strands of literature. First, it relates to the use of text to define markets and product spaces. Gentzkow et al. (2019) survey the methods, and Ash and Hansen (2023) review later algorithms. I build on Hoberg and Phillips (2016) and Eisfeld (2025) for continuous market definitions from filings and software descriptions. For patent text, the closest work is topic modeling of recombination and diversification (Kaplan and Vakili, 2015; Teodoridis et al., 2022), breakthrough measurement from the text itself (Kelly et al., 2021), similarity and novelty from claims (Arts et al., 2018, 2021), joint patent-job-earnings text (Kalyani et al., 2025), transformer embeddings (Hain et al., 2022), and text measures of corporate innovation from analyst reports (Bellstam et al., 2021). I translate those textual outputs into state variables for a structural entry model.

Second, the paper contributes to work on the valuation and use of intellectual property. I use the grant-window valuations of Kogan et al. (2017). Hall et al. (2005) measure related objects with citations. Related evidence includes defensive patenting by market leaders (Argente et al., 2020), internal versus external innovation by firm size (Akcigit and Kerr, 2018), citation responses to invalidation (Galasso and Schankerman, 2015), semiconductor thickets (Shapiro, 2001; Ziedonis, 2004), and mega-firm patenting (Choi et al., 2025). I use those valuations to parameterize expected profit from technological expansion.

Third, the methodology follows structural entry and partial identification. Classical designs recover competition from market structure (Bresnahan and Reiss, 1991; Berry, 1992; Mazzeo, 2002; Seim, 2006). Multiple equilibria force set inference (Tamer, 2003; Ciliberto and Tamer, 2009; Ciliberto et al., 2021). Discrete games via inequalities follow Pakes (2010), Pakes et al. (2015), Pakes and Porter (2024a), Kline et al. (2021), and Magnolfi and Roncoroni (2023). Combinatorial multi-market applications include Jia (2008), Holmes (2011), Ellickson et al. (2013), Eizenberg (2014), and Wollmann (2018). The add, drop, and swap deviations follow Morales et al. (2019) and Dickstein and Morales (2018); the same extensive margin appears in sourcing and multinational production (Antràs et al., 2017; Tintelnot, 2017; Arkolakis et al., Forthcoming, 2018; Gentzkow, 2007). I apply that algebra to a complete-information aggregative game of area entry, with congestion in rival counts and a scope term in portfolio size. Inference uses the many-moment inequalities of Chernozhukov et al. (2019), with guidance from Canay and Shaikh (2017) and Canay et al. (2023). Pakes and Porter (2024b) develop related inequalities for multinomial choice with fixed effects. The game is static. Dynamic oligopoly studies of electronics recover preemption and option value (Goettler and Gordon, 2011; Igami, 2017; Igami and Uetake, 2020; Chen and Xu, 2023). Those objects require a continuation value, a transition for rival counts, and an equilibrium-selection rule, which I do not impose.

Finally, the findings speak to the limits of the firm and to technology spillovers. Jaffe (1986) and Bloom et al. (2013) measure spillovers in technology space. Aghion et al. (2005) document an inverted-U between competition and innovation, and Hall and Ziedonis (2001) document the semiconductor patenting surge. Once expected valuations are held fixed, additional rivals reduce profit. On scope, related work includes pharmaceutical research (Henderson and Cockburn, 1996), product-line dynamics (Klette and Kortum, 2004), recombination (Weitzman, 1998; Fleming, 2001), relatedness and diversity (Breschi et al., 2003; Miller, 2006), and multi-location research (Chikis et al., 2025). The bounds recover synergies that rise with portfolio size. A high per-area fixed cost together with rival congestion limits expansion. Rotemberg and Saloner (1994), Seru (2014), and Rawley and Simcoe (2010) stress agency and overload costs. The local cost-reduction exercises speak to industrial policy (Juhász et al., 2024; Goldberg et al., 2024; Barwick et al., 2025; Bloom et al., 2019). I hold rival counts at their observed values. The reported ranges are partial-equilibrium reallocations at those counts.

2 Data, Methodology, and Stylized Facts

2.1 Data

I construct a comprehensive firm-level panel covering the U.S. electronics industry from 1990 to 2019. Financial and organizational characteristics, primarily total assets and R&D expenses, are sourced from the Compustat dataset. I constrain the sample to publicly listed firms categorized under the two-digit Standard Industrial Classification (SIC) code 36 (Electronics and Electrical

Components). To ensure panel integrity, I require consecutive yearly observations, effectively screening out firms with disjointed dataset appearances, and impute missing R&D and asset values with zero. Total assets serve as the proxy for firm size, aligning with standard empirical finance conventions (Seru, 2014). This yields a final sample of 16,653 firm-year observations distributed across 1,373 unique firms. Of these, 755 firms have at least one patent during the sample.

To capture the economic value and categorical nature of these firms' innovations, I integrate the patent dataset from Kogan et al. (2017), which provides market-implied patent valuations (derived from stock price movements subsequent to patent issuances), forward citation counts, and primary classifications. The three-day grant-window return recovers the unanticipated increment to private value, scaled by the grant-probability factor in Kogan et al. (2017). It is the value of the grant announcement. The LDA weights then allocate that grant-level increment across topics. I link this intellectual property data to the Compustat financial panel utilizing the dynamic identifier correspondence tables (permco/permno to gvkey) provided by Wharton Research Data Services (WRDS)[2].

The sample is the intersection of Compustat electronics firms and the Kogan et al. (2017) valuations. Those valuations require a listed firm with a traded return around the patent grant date, so startups and privately held innovators are outside the panel by construction. The two-digit SIC 36 extract also omits diversified firms whose electronics patents sit in another industry code, and non-electronics firms that enter these topics. Of the 1,373 sample firms, 755 have at least one patent. The entry indicator, realized valuations, and their first-stage projection are defined on patenting firm-years. The remaining firms appear in the Compustat extract but not in the entry sample. They are not recorded as occupying zero areas. A patenting firm-year can still occupy zero areas if every topic weight is below the 1 percent floor. Those years still speak to why empty areas stay empty. The scope benefit is identified from firm-years that occupy several areas at once.

This selection has a signed reading for the objects estimated later. Listed electronics firms face heavier disclosure, compliance, and scale requirements than a typical startup, so the estimated per-area entry cost is likely larger than the cost that would apply to the broader innovator population. Multi-area bundling is also the comparative advantage of large listed firms, so the estimated scope benefit is likely stronger than it would be for a single-product entrant. The sign of congestion is a market-level object and is more likely to travel. Its magnitude need not travel.

Because structural topic modeling requires rich textual inputs, I augment the Kogan et al. (2017) data by collecting unstructured text, specifically patent titles and abstracts, directly from Google Patents. My final textual corpus consists of 631,369 patents covering the electronics industry across

[2]For an extensive discussion detailing the nuances of dynamically matching Compustat and USPTO entities, see Arora et al. (2021) and Dyevre and Seager (2023).

the timeline[3].

Table 1 summarizes the firm characteristics I use.

Table 1: Summary statistics of firm characteristics

Variable	Statistic	Note
Number of firms (full sample)	1,373 (100%)	
Firms with at least one patent	755 (55%)	
R&D (millions USD, real 2012)	161.39	Average for patenting firms
Assets (millions USD, real 2012)	2663.77	Average for patenting firms
Patents (flow, annual)	54.16	Average annual patents
Patents (stock, cumulative)	4568.41	Average cumulative patents

2.2 Methodology: Topic Modeling

To parse the vast textual corpus of electronics patents into economically meaningful dimensions, I employ Latent Dirichlet Allocation (LDA; Blei et al., 2003), treating each patent as a probabilistically weighted mixture of underlying ”topics” or innovation areas. Conceptually, this mirrors the clustering methodologies frequently utilized in the industrial organization literature to define product market segments based on business descriptions (e.g., Eisfeld, 2025), but it applies a sophisticated generative probabilistic model engineered entirely for unstructured text.

Administrative patent classes such as the International Patent Classification and the Cooperative Patent Classification assign each grant to examiner-chosen nodes. Those nodes are discrete, historically path dependent, and do not supply continuous weights when a patent lists several codes. LDA recovers a probability distribution over areas from the patent text, so a single invention can span several domains with explicit weights. I select the number of topics and the Dirichlet priors by a five-fold grid search that balances perplexity, NPMI coherence, exclusivity, and decade-to-decade stability. Appendix A.1 through Appendix A.4 record the preprocessing, the search, and the metrics.

To assign labels, I extract the most heavily weighted terms for each area and iterate between two large language models (OpenAI’s GPT-o3 and xAI’s Grok 3 Think) until the proposed names converge. Table 2 lists the twenty labels. Top words are in Appendix A.5.

Translating the raw LDA structural assignments into empirically tractable components for the structural model requires aggregating from the patent level to the firm-year level. By dropping trivial topic exposures (weights below 1%), I construct the following key variables tailored to the entry model framework: (i) A monetary valuation of the patent and of the firm within each innovation area, A_{ipt} and v_{ijt}, along with the LDA weights, w_{pjt}, at the patent level. (ii) A binary

[3] A negligible fraction of patents (0.5%) were omitted from the final corpus due to missing records or irretrievable web elements.

Table 2: Innovation areas

Topic	Label	Topic	Label
1	Semiconductor Memory Architecture	11	Power-Supply and Power-Management Electronics
2	Audio and Speech-Signal Processing	12	Networking Protocols and Data Infrastructure
3	Semiconductor Lithography and Fabrication Processes	13	Transistor Gate Structures and Semiconductor Device Physics
4	Integrated Circuit Design, Verification, and Testing	14	Imaging Sensors and Camera Pipelines
5	PCB and Electronic Hardware with Connectors	15	Processor Architecture, Caches, Instruction Execution
6	Video Recording, Encoding, and Media Processing	16	Display Panel Tech (LCD/OLED)
7	Semiconductor Packaging and Interconnect Bonding	17	Rotational Electromechanics and Magnetic Actuation
8	Thin-Film Materials, Deposition and Chemistry	18	Optics, Lasers, and Photonics
9	Wireless Base-Station and Mobile Communications	19	Analog and Differential Amplifier Circuits
10	Clock Generation, Timing Circuits, and Signal Processing	20	Mobile UI and Content Services

Notes: Labels are assigned from the twenty-topic LDA fit in Section 2.2. Top words for each area are in Appendix A.5.

indicator of firm i's entry into innovation area j at time t, d_{ijt}, which equals 1 if the firm has sufficient topic presence, and 0 otherwise. (iii) A congestion metric capturing the aggregate number of competing firms residing within area j, $N_{jt,-i} \equiv \sum_{i' \neq i} d_{i'jt}, \forall j, t$. (iv) The number of areas the firm occupies in each period, $k_{it} := |J_{it}| = \sum_j d_{ijt}$. (v) Predetermined firm stocks used later as instruments: lagged R&D, RD_{it-1}, total assets, AT_{it-1}, and patent flow, PAT_{it-1}. These systematically constructed variables serve as the foundational inputs for the structural estimation introduced in the subsequent section. The 1 percent floor defines whether the firm is in the area. Intensity of occupation, given entry, is a different outcome.

2.3 Stylized Facts

I provide a concise set of stylized facts detailing the evolutionary dynamics of the U.S. electronics industry across these empirically derived innovation areas. I leverage aggregated firm and topic-level statistics to highlight the heterogeneity across technological domains. These facts establish the foundational empirical regularities, specifically regarding combinatorial innovation, competitive congestion, and technological rotation, that the structural model will explicitly seek to rationalize.

Stylized Facts 1 (On Multi-Domain Bundling and Portfolio Scope) *Technological boundaries within the electronics industry are remarkably porous. Innovation rarely occurs in isolated silos; instead, inventive efforts and organizational entry persistently span multiple distinct technological domains simultaneously.*

Tables 3 and 4 demonstrate that multi-domain bundling is a dominant, defining feature of elec-

Table 3: Patent-level summary statistics

Variable	Min	Q25	Q75	Max	Median	Mean	SD
w_{pjt} (LDA weight)	0	0	0.001	1	0	0.05	0.154
A_{ipt} (Patent value, million USD)	0	0.006	0.169	39.947	0.052	0.184	0.506
k_{ipt} (Number of areas per patent)	0	3	4	11	3	3.583	1.387

Table 4: Firm-year summary statistics

Variable	Min	Q25	Q75	Max	Median	Mean	SD
d_{ijt} (Entry indicator)	0	0	1	1	0	0.269	0.443
$N_{jt,-i}$ (Competitor entry count)	20	83	141	218	105	110.255	39.913
k_{it} (Number of areas firm enters)	0	0	10	20	5	5.373	5.429
$\sum_p w_{pjt} A_{ipt}$ (Patent value sum, million USD)	0	0	0.721	112263.324	0	51.205	743.379

tronics innovation. At the patent level (Table 3), the average patent functionality spans 3.58 distinct innovation areas (k_{ipt}), indicating that a single invention relies heavily on combinatorial knowledge. Scaling up to the firm's organizational level (Table 4), I find that firm-years command broad innovation portfolios. The median firm-year occupies 5 distinct areas (k_{it}), with the top quartile participating in 10 or more domains simultaneously. These statistics show that entry is a joint portfolio choice, subject to economies or diseconomies of scope.

Stylized Facts 2 (On Competitive Congestion and Market Structure) *The density of competitors is highly asymmetric across innovation areas. While certain thematic domains reflect heavily congested, low-barrier ecosystems, others maintain structurally constrained innovator bases over decades.*

Table 5 traces the share of active innovators in each area. Foundational domains, such as "Integrated Circuit Design, Verification, and Testing" (Topic 4) and "Audio and Speech-Signal Processing" (Topic 2), routinely absorb about 7 percent of the industry's active base. "Display Panel Tech (LCD/OLED)" (Topic 16) remains sparse, under 2 percent throughout. Appendix A.6 reports the corresponding counts.

Stylized Facts 3 (On Valuation Dynamics and Technological Rotation) *The composition of active firms, and the economic valuation of patents, rotate away from hardware and materials engineering toward mobile, wireless, and interface technologies.*

Table 6 reports the share of industry patent value in each area. Table 5 shows that legacy hardware such as "Rotational Electromechanics and Magnetic Actuation" (Topic 17) and "Thin-Film Materials, Deposition and Chemistry" (Topic 8) shed competitor share from the 1990s through the 2010s, while "Semiconductor Packaging and Interconnect Bonding" (Topic 7), "Wireless Base-Station and Mobile Communications" (Topic 9), and "Networking Protocols and Data Infrastructure" (Topic 12) gain. In the 1990s, "Rotational Electromechanics and Magnetic Actuation" (Topic 17) and "Thin-Film Materials, Deposition and Chemistry" (Topic 8) together accounted for more

Table 5: Composition of competitors by topic and decade

Topic	Label	1990s (%)	2000s (%)	2010s (%)	Mean (%)	SD
1	Semiconductor Memory Architecture	3.76	3.37	2.94	3.41	0.41
2	Audio and Speech-Signal Processing	6.97	7.19	7.1	7.08	0.23
3	Semiconductor Lithography and Fabrication Processes	4.39	4.33	4.27	4.34	0.32
4	Integrated Circuit Design, Verification, and Testing	7.49	7.68	7	7.44	0.41
5	PCB and Electronic Hardware with Connectors	6.97	6.47	6.73	6.72	0.4
6	Video Recording, Encoding, and Media Processing	3.7	3.73	3.35	3.62	0.31
7	Semiconductor Packaging and Interconnect Bonding	3.77	4.61	5.18	4.44	0.64
8	Thin-Film Materials, Deposition and Chemistry	6.31	5.14	4.66	5.45	0.79
9	Wireless Base-Station and Mobile Communications	3.62	4.35	4.74	4.18	0.53
10	Clock Generation, Timing Circuits, and Signal Processing	6.87	6.83	6.4	6.74	0.34
11	Power-Supply and Power-Management Electronics	5.97	6.14	6.27	6.11	0.28
12	Networking Protocols and Data Infrastructure	3.54	4.36	4.45	4.08	0.54
13	Transistor Gate Structures and Semiconductor Device Physics	3.6	3.78	3.72	3.7	0.2
14	Imaging Sensors and Camera Pipelines	3.12	3.3	4.22	3.46	0.54
15	Processor Architecture, Caches, Instruction Execution	5.1	5.47	5.53	5.35	0.33
16	Display Panel Tech (LCD/OLED)	1.86	1.81	1.92	1.85	0.19
17	Rotational Electromechanics and Magnetic Actuation	7.39	5.82	5.95	6.43	0.87
18	Optics, Lasers, and Photonics	3.84	3.78	3.91	3.84	0.22
19	Analog and Differential Amplifier Circuits	6.24	6.12	5.9	6.11	0.28
20	Mobile UI and Content Services	5.51	5.73	5.78	5.66	0.26
	Total	100.02	100.01	100.02	100.01	–

Table 6: Composition of patent valuation by topic and decade

Topic	Label	1990s (%)	2000s (%)	2010s (%)	Mean (%)	SD
1	Semiconductor Memory Architecture	2	2.12	1.62	1.94	5.15
2	Audio and Speech-Signal Processing	6.42	7.1	6.99	6.84	10.98
3	Semiconductor Lithography and Fabrication Processes	3.27	3.65	3.74	3.55	7.91
4	Integrated Circuit Design, Verification, and Testing	5.66	6.12	5.14	5.7	8.43
5	PCB and Electronic Hardware with Connectors	10.2	8.73	8.19	9.08	16.36
6	Video Recording, Encoding, and Media Processing	2.17	2.16	2	2.12	5.38
7	Semiconductor Packaging and Interconnect Bonding	2.42	4.4	4.96	3.88	9.98
8	Thin-Film Materials, Deposition and Chemistry	9.42	7.2	5.89	7.6	16.49
9	Wireless Base-Station and Mobile Communications	2.34	3.02	4.14	3.1	7.45
10	Clock Generation, Timing Circuits, and Signal Processing	7.76	7.85	6.79	7.53	11.91
11	Power-Supply and Power-Management Electronics	4.71	5.31	6.09	5.32	10.53
12	Networking Protocols and Data Infrastructure	3.4	5.2	5.19	4.58	10.87
13	Transistor Gate Structures and Semiconductor Device Physics	2.59	3.38	3.73	3.21	8.51
14	Imaging Sensors and Camera Pipelines	1.98	2.06	3.06	2.31	5.83
15	Processor Architecture, Caches, Instruction Execution	4.57	4.46	5	4.64	9.42
16	Display Panel Tech (LCD/OLED)	1.07	1.35	1.33	1.25	4.83
17	Rotational Electromechanics and Magnetic Actuation	14.14	8.85	8.39	10.52	18.68
18	Optics, Lasers, and Photonics	3.4	3.98	4.08	3.81	9.45
19	Analog and Differential Amplifier Circuits	7.19	7.38	6.22	7	12.44
20	Mobile UI and Content Services	5.28	5.68	7.45	6.03	12.5
	Total	99.99	100	100	100.01	–

than 23 percent of patent value. By the 2010s their combined share had fallen, and value had moved toward "Mobile UI and Content Services" (Topic 20), "Wireless Base-Station and Mobile Communications" (Topic 9), and "Power-Supply and Power-Management Electronics" (Topic 11). Topic 20's share rose from 5.28 percent in the 1990s to 7.45 percent in the 2010s. Appendix A.6 reports the corresponding dollar levels.

3 Model and Implementation

To explain the stylized facts from the previous section, I develop a simple entry model following Pakes (2010), Pakes et al. (2015), Pakes and Porter (2024a), Dickstein and Morales (2018), and Morales et al. (2019). For empirical implementation, I adopt the moment inequalities approach of Canay and Shaikh (2017), Chernozhukov et al. (2019), and Canay et al. (2023).

3.1 Information Structure and The Game

I model the firm's decision to enter innovation areas as a static, simultaneous-move, aggregative game under symmetric uncertainty, or complete information over observable state variables. When deciding to enter area j in period t, firm i uses its *ex-ante* information set $\mathcal{F}_{ijt}$ to evaluate expected profits. That set contains publicly observable states, including the firm's own R&D stock, its past patenting in specific areas, and characteristics of rivals, which the econometrician observes only in part. Unobservable area-specific market shocks u_{ijt} realize after entry, so $E[u_{ijt} \mid \mathcal{F}_{ijt}] = 0$. Firms therefore play a simultaneous-move Nash equilibrium over expected payoffs. Congestion $N_{jt,-i}$ is evaluated at the observed rival count.

This information structure follows Dickstein and Morales (2018) and Morales et al. (2019): firms do not select on shocks they have not yet seen, which delivers linear moment inequalities. The static game answers how a firm composes its contemporaneous portfolio and how rival counts in the same period shift expected profit. If firms enter in part for option value, estimated κ absorbs that option and is an upper bound on the purely contemporaneous fixed cost. Appendix B.3 records the remaining dynamic caveats.

3.2 Firm Valuation and Expected Profit

Each patent p filed by firm i in period t has a dollar value A_{ipt}. Using topic modeling from the prior section, weights w_{pjt} (where $0 \leq w_{pjt} \leq 1$ and $\sum_j w_{pjt} = 1$) allocate each patent across innovation areas. The area-specific value is $A_{ipjt} = w_{pjt} A_{ipt}$ [4].

The *realized ex-post* firm valuation is then

$$v_{ijt} = b \left(\sum_{p \in \mathcal{P}_{it}} w_{pjt} A_{ipt} \right) + e_{ijt},$$

[4] The construction of A_{ipt} is in Appendix B.1. The construction of the LDA weights w_{pjt} is in Appendix A.2.

where $\mathcal{P}_{it}$ is firm i's patent set in period t, and $b(\cdot)$ is a monotonically increasing transformation function. This function maps the raw aggregated patent values into an empirically tractable scale, managing the high skewness typical of innovation data to make the measure suitable for estimation. The term e_{ijt} captures valuation measurement error, explicitly absorbing the noise inherent to LDA topic assignments and the stock market's reaction in the event studies. Firms do not observe the exact realized error e_{ijt} prior to entry, meaning their ex-ante expectation is strictly deterministic: $E[e_{ijt} \mid \mathcal{F}_{ijt}] = 0$. The expected valuation is $\check{v}_{ijt} \equiv E[v_{ijt} \mid \mathcal{F}_{ijt}]$.

The true realized entry profit for firm i into area j in period t depends on this actual valuation alongside competitive and organizational factors:

$$\pi_{ijt} = v_{ijt} + \alpha h(N_{jt,-i}) + \beta f(k_{it}) - \kappa + u_{ijt}$$

By substituting the expected valuation framework, $v_{ijt} = \check{v}_{ijt} + e_{ijt}$, into the true profit function, the estimating equation is:

$$\pi_{ijt} = \check{v}_{ijt} + \alpha h(N_{jt,-i}) + \beta f(k_{it}) - \kappa + \epsilon_{ijt}$$

where $N_{jt,-i}$ is the number of competitors (excluding the firm itself) in area j, and $k_{it} \equiv |J|_{it}$ is the total number of innovation areas firm i is active in during period t. $\check{v}_{ijt} \equiv E\left[b\left(\sum_p w_{pjt} A_{ipt}\right) \mid \mathcal{F}_{ijt}\right]$ is the expected firm valuation, and $h(\cdot)$ and $f(\cdot)$ denote deterministic transformation functions. The unobserved error term in the estimating equation, $\epsilon_{ijt} \equiv e_{ijt} + u_{ijt}$, is a *composite ex-post forecasting error*. It absorbs both the valuation measurement noise (e_{ijt}) and the market shocks (u_{ijt}). Because both components are mean-zero conditional on the firm's information set, $E[\epsilon_{ijt} \mid \mathcal{F}_{ijt}] = 0$.

Parameter α captures competition effects: positive values indicate spillovers (e.g., net-agglomeration effect), while negative values reflect crowding out. Parameter β measures the impact of portfolio diversity, with positive values suggesting benefits from diversification (e.g., economies of scope) and negative values indicating costs of overextension (e.g., diluted focus). Parameter κ is a per-area fixed entry cost. Let $\theta = (\alpha, \beta, \kappa)$ be the vector of parameters of interest.

Let d_{ijt} be an indicator whether firm i enters into area j at time t. The firm chooses a bundle of areas to maximize expected total profit:

Result 1 (Firm's Expected Total Profit) *Because $E[\epsilon_{ijt} \mid \mathcal{F}_{ijt}] = 0$, the expected total profit simplifies to its deterministic components:*

$$\begin{aligned}\mathbb{E}[\Pi_{it} \mid \mathcal{F}_{ijt}] &= \sum_{j=1}^{J} d_{ijt}\mathbb{E}[\pi_{ijt} \mid \mathcal{F}_{ijt}] = \sum_{j=1}^{J} d_{ijt}\left(\check{v}_{ijt} + \alpha h(N_{jt,-i}) - \kappa\right) + \beta f(k_{it}) \sum_{j=1}^{J} d_{ijt} \\ &= \sum_{j \in S}\left(\check{v}_{ijt} + \alpha h(N_{jt,-i}) - \kappa\right) + \beta g(k_{it})\end{aligned}$$

where $S = \{j \mid d_{ijt} = 1\}$, $k_{it} = |J|_{it}$, and $g(k_{it}) \equiv k_{it} f(k_{it})$. Result 1 highlights the importance of the bundling problem: the presence of the portfolio term $\beta g(k_{it})$ creates interdependencies across choices[5].

Firm i selects the bundle $\mathbf{d}_{it} = (d_{ijt})_j \in \{0,1\}^J$ to maximize expected profit $\mathbb{E}[\Pi_{it} \mid \mathcal{F}_{ijt}]$ in a simultaneous-move game. Solving for the full equilibrium fixed point over a strategy space of 2^J bundles is computationally intractable due to the curse of dimensionality. However, in any Nash Equilibrium, the firm's observed bundle must be its optimal best response, taking the equilibrium actions of rival firms ($N_{jt,-i}$) as given.

Because the portfolio cost depends only on the *count* of entered areas (k_{it}) and not their specific identities, the firm's complex combinatorial optimization problem simplifies significantly conditional on these rival actions. This yields a characterization of the firm's optimal bundle:

Result 2 (Characterization of the Optimal Bundle) *The expected profitability index for area j is:*

$$\zeta^e_{ijt} = \check{v}_{ijt} + \alpha h(N_{jt,-i}) - \kappa$$

For a fixed portfolio size k_{it}, the firm maximizes profit by choosing the k_{it} areas with the largest ζ^e_{ijt}. That is, the areas are sorted such that: $\zeta^e_{(1)} \geq \zeta^e_{(2)} \geq \cdots \geq \zeta^e_{(J)}$. Then, the optimized expected profit for a portfolio of size k is:

$$U^e_{k_{it}} = \sum_{m=1}^{k_{it}} \zeta^e_{(m)} + \beta g(k_{it})$$

An optimal size is any $k^*_{it} \in \arg\max_{k_{it}=0,1,\dots,J} U^e_{k_{it}}$. Any optimal bundle is a set of k^*_{it} areas with the largest ζ^e_{ijt}, allowing ties at the cutoff, and $d_{ijt} = 1$ if and only if j belongs to that bundle.

Proof. See Appendix B.2. ■

Assuming the portfolio cost $\beta g(k_{it})$ depends only on the count k_{it} vastly simplifies the firm's optimization problem: the firm ranks areas by ζ and selects a top-k^*_{it} set. However, it does not solve the econometrician's estimation problem. The decision to enter area j is still not separable from area m[6]. Furthermore, although Result 2 provides a clean characterization, point identification of θ remains difficult due to multiple equilibria and combinatorial explosion. Which configuration materializes depends on an unobserved equilibrium-selection rule (as shown by Tamer, 2003; Ciliberto and Tamer, 2009).

[5]A standard independent single-agent discrete choice framework is ill-posed in this setting because the portfolio term $\beta g(k_{it})$ renders the entry decisions non-separable; the marginal payoff of entering area j explicitly depends on the endogenous decision to enter area m. Consequently, the firm is not making J independent choices, but rather selecting one bundle out of 2^J possible combinations. With $J = 20$ innovation areas, evaluating standard choice probabilities over 2^{20} (more than one million) alternative bundles per firm-year is computationally intractable, necessitating the moment inequalities approach.

[6]The marginal cost of entering j depends on k_{it}, which is endogenously determined by the entry decisions for all other areas. Because the choices are jointly determined, those decisions cannot be estimated as J independent single-choice models (e.g., in typical logit). The outcome is a single chosen bundle S^*.

Instead, I pursue partial identification following Pakes (2010), Pakes et al. (2015), and Pakes and Porter (2024a). This replaces unattainable moment equalities with inequalities satisfied by every equilibrium. Based on Result 2, any deviation from the observed choice S^* must yield a lower (or equal) expected profit. I consider 'step and swap deviations': (i) entering M additional areas, (ii) exiting M current areas, or (iii) swapping the entry portfolio while keeping k_{it} constant.

Let $Z_{ijt} \in \mathbb{R}^n$ be a vector of instruments observable to both the econometrician and the firm, assumed to be a subset of $\mathcal{F}_{ijt}$. Let $l : \mathbb{R}^n \to \mathbb{R}_+$ map Z_{ijt} to a non-negative scalar. Following Canay et al. (2023), these instruments are weights that partition the sample and tighten the inequalities. They do not instrument rival counts $N_{jt,-i}$ or portfolio size k_{it} in the two-stage least squares sense. Common area-time attractiveness is absorbed in the first-stage projection of expected value. Swaps then compare two areas at fixed k_{it}.

First, I will derive the moment functions for adding M areas.

Result 3 (Moment Functions for Adding M Areas (Non-Entrant Deviation)) *Let $\mathcal{M}$ be an exogenously drawn subset of M areas ($M = |\mathcal{M}|$). I consider the event where the firm chose to enter none of the areas in $\mathcal{M}$, indicated by $D^0_{\mathcal{M},it} \equiv \prod_{j\in\mathcal{M}}(1 - d_{ijt}) = 1$. The difference in expected profit between the deviation (adding $\mathcal{M}$) and the observed choice is, $\mathbb{E}[\Delta\Pi_{it} \mid \mathcal{F}_{ijt}]$:*

$$\mathbb{E}[\Delta\Pi_{it} \mid \mathcal{F}_{ijt}] = \sum_{j\in\mathcal{M}} \left(\check{v}_{ijt} + \alpha h(N_{jt,-i}) - \kappa\right) + \beta\Delta g^+_M(k_{it}) = \eta^+_{i,\mathcal{M},t}$$

where $\Delta g^+_M(k_{it}) = g(k_{it} + M) - g(k_{it})$. If the firm chose not to enter any area in $\mathcal{M}$, the expected deviation profit must be non-positive: $\eta^+_{i,\mathcal{M},t} \leq 0$. Multiplying this by the choice indicator and the non-negative instrument yields the unconditional moment function:

$$\mathbf{M}_1(Z_{i,\mathcal{M},t};\theta) = E\left[D^0_{\mathcal{M},it} \cdot \eta^+_{i,\mathcal{M},t} \cdot l(Z_{i,\mathcal{M},t})\right] \leq 0 \tag{1}$$

Proof. See Appendix B.2. ■

Now, I will derive the moment functions for dropping M areas.

Result 4 (Moment Functions for Dropping M Areas (Entrant Deviation)) *Let $\mathcal{M}$ be an exogenously drawn subset of M areas ($M = |\mathcal{M}|$). I consider the event that the firm entered every area in $\mathcal{M}$, indicated by $D^1_{\mathcal{M},it} \equiv \prod_{j\in\mathcal{M}} d_{ijt} = 1$. The expected profit difference of dropping $\mathcal{M}$ is:*

$$\mathbb{E}[\Delta\Pi_{it} \mid \mathcal{F}_{ijt}] = -\sum_{j\in\mathcal{M}} \left(\check{v}_{ijt} + \alpha h(N_{jt,-i}) - \kappa\right) - \beta\Delta g^-_M(k_{it}) = -\eta^-_{i,\mathcal{M},t}$$

where $\Delta g^-_M(k_{it}) = g(k_{it}) - g(k_{it} - M)$. If the firm chose to enter all areas in $\mathcal{M}$, dropping them cannot yield strictly higher expected profit. Thus, $-\eta^-_{i,\mathcal{M},t} \leq 0$. The unconditional moment function evaluates as:

$$\mathbf{M}_2(Z_{i,\mathcal{M},t};\theta) = E\left[D^1_{\mathcal{M},it} \cdot (-\eta^-_{i,\mathcal{M},t}) \cdot l(Z_{i,\mathcal{M},t})\right] \leq 0 \tag{2}$$

Proof. See Appendix B.2. ■

These M-step deviations $\mathbf{M}_1$ and $\mathbf{M}_2$ moments are all valid and necessary conditions, yet they do not fully exploit the sorting characterization outlined in Result 2. To avoid endogenous selection bias, I define swap moments over exogenously chosen pairwise deviations. Because the portfolio term depends solely on the count k_{it}, any profitable swap of an entered area for a non-entered area, while keeping k_{it} fixed, is ruled out. I can then derive this 'swap' moment function as follows.

Result 5 (Moment Functions for Pairwise Swap Deviation) *Let (j, m) be an ordered pair where firm i chose to enter j but not m: $D^{swap}_{j,m,it} \equiv d_{ijt}(1 - d_{imt}) = 1$. The expected profit of deviating by substituting m for j leaves the portfolio size constant ($\Delta g = 0$):*

$$\mathbb{E}[\Delta\Pi_{it} \mid \mathcal{F}_{ijt}] = (\check{v}_{imt} + \alpha h(N_{mt,-i})) - \left(\check{v}_{ijt} + \alpha h(N_{jt,-i})\right) = \mu_{ijmt}(\theta)$$

Optimality requires that the expected swap profit is non-positive: $\mu_{ijmt}(\theta) \leq 0$. The unconditional linear moment function is:

$$\mathbf{M}_3(Z_{ijmt}; \theta) = E\left[D^{swap}_{j,m,it} \cdot \mu_{ijmt}(\theta) \cdot l(Z_{ijmt})\right] \leq 0 \tag{3}$$

Proof. See Appendix B.2. ■

The swap moment, $\mathbf{M}_3$, isolates the identification of the congestion parameter α, free from the portfolio scale effect. Common area-time attractiveness is absorbed earlier. The first-stage projection includes firm and year-by-area fixed effects, so a year in which an area is attractive to every firm is already in $\check{v}_{ijt}$. After that projection, a swap compares two areas for the same firm-year. Portfolio size is held fixed, the $\beta g(k)$ term cancels, and the remaining comparison is $\check{v}_j + \alpha h(N_j)$ against $\check{v}_m + \alpha h(N_m)$. The residual variation that identifies α is therefore whether the firm occupies the more crowded of two areas that look similar in expected value, and whether it still enters a crowded area only when expected value covers the per-area cost. Add and drop moments then pin the level of κ and β. Parameter α remains a net effect of business stealing, litigation, and any residual unobserved attractiveness that the first stage did not absorb.

3.3 Empirical Implementation

Entry is $d_{ijt} = \mathbf{1}\{\frac{1}{|\mathcal{P}_{it}|}\sum_{p \in \mathcal{P}_{it}} w_{pjt} \geq 0.01\}$. Weights below 1 percent are the long tail of the topic model. The cutoff defines the discrete object of the game.

I recover $\check{v}_{ijt}$ by applying the inverse hyperbolic sine to realized v_{ijt} and projecting onto lagged R&D stocks, assets, and patent flow, together with firm and year-by-area effects. The same three stocks, coded as upper-tier indicators, weight the inequalities. Following Canay et al. (2023), those indicators split the sample. They are not excluded instruments for rival counts or portfolio size.

Congestion enters as $h(N) = \operatorname{asinh}(N)$, which is steep in thin markets and flatter once an area is crowded. The portfolio term is $g(k) = \binom{k}{2}$, which counts pairs among entered areas and delivers the sorting in Result 2. I omit single-area add and drop moments because $\Delta g_1^+ = k_{it}$ and $\Delta g_1^- = k_{it} - 1$ are nearly collinear with κ. Pair and triplet deviations break that collinearity.

I rank candidate add and drop subsets from predicted values $\check{v}_{ijt}$ before looking at realized d_{ijt}. For drop moments I keep the subsets with the highest average predicted value. For add moments I keep those with the lowest. For swaps I rank the 380 ordered pairs by divergence in $\check{v}$ and draw from the top, the bottom, and the middle of that ranking. Table 9 varies how many of those ranked subsets enter the inequalities ($|M|$). The reported sets use samples of 20 or 40 per deviation type, and the specification with all three instruments uses the largest of those samples. The object is a 99 percent confidence set for θ. The test statistic is the max-type studentized sample moment of Chernozhukov et al. (2019), with their two-step self-normalized critical value at significance level 0.01. Appendix B.3 records the search over a bounded parameter region and the profiled bounds.

4 Results

I first present the first-stage projection required to recover the firm's *ex-ante* valuations from noisy *ex-post* realizations. Next, I report a reduced-form linear probability model of entry. Finally, I present the estimated identified sets from the moment inequalities, which isolate congestion, scope, and the per-area cost.

4.1 First-stage Projection: Extracting Expected Valuations

The raw aggregated patent value, $v_{ijt} = b\left(\sum_{p \in \mathcal{P}_{it}} w_{pjt} A_{ipt}\right)$, is an *ex-post* realization inherently plagued by structural measurement errors. These errors originate from three primary sources: (i) Latent Dirichlet Allocation (LDA) misclassification, which introduces random probabilistic noise into the structural weights w_{pjt}; (ii) stochastic noise in market-implied patent valuations, A_{ipt}, derived from event study stock reactions (Kogan et al., 2017); and (iii) the mechanical aggregation frictions of mapping patent-level text distributions to the firm-year-topic level. If v_{ijt} is directly inserted into the structural entry framework, classical measurement error would attenuate the valuation term, which is the scale of the profit function. The first-stage projection recovers the component of that realized value that is predictable from the firm's information set.

To purge this measurement error and properly recover the firm's *ex-ante* expected valuation, $\check{v}_{ijt} \equiv E[v_{ijt} \mid \mathcal{F}_{ijt}]$, I project the observed v_{ijt} onto a vector of predetermined firm-state instruments $Z_{ijt} = (\mathrm{RD}_{it-1}, \mathrm{AT}_{it-1}, \mathrm{PAT}_{it-1})$ alongside rich fixed effects configurations. The fixed effects systematically soak up unobserved heterogeneity (such as macroeconomic shifts or area-specific hype cycles), while covariates capture the predictable variation governed by the firm's past scale and R&D investments. Consequently, the residual noise, which encompasses both true expectational forecasting and aforementioned measurement errors, is averaged out asymptotically.

As formalized in the empirical implementation section, the state variables Z_{ijt} are discretized into binary indicators to efficiently manage the extreme skewness of innovation data. Specifically, past R&D stocks (RD_{it-1}), total assets (AT_{it-1}), and past patent flow (PAT_{it-1}) take a value of 1 for firm-year observations in the upper tier (medium-to-high levels, denoted as bin 2). The lower tier (zero or bottom-quartile, bin 1) serves as the baseline reference category.[7] Table 7 reports the first-stage projection, demonstrating that these observable proxies possess strong, positive predictive power.

Table 7: First Stage: Expected Profit Projections

Dependent Variable:	$v_{ijt} = b(\sum_p w_{pjt} A_{ipt})$		
Model:	(1)	(2)	(3)
Variables			
Constant	-0.464***		
	(0.006)		
RD_{it-1}	0.320***	0.072***	0.072***
	(0.022)	(0.021)	(0.021)
AT_{it-1}	0.206***	0.109***	0.109***
	(0.021)	(0.021)	(0.021)
PAT_{it-1}	1.90***	0.837***	0.837***
	(0.130)	(0.072)	(0.072)
Fixed-effects			
Firm		Yes	Yes
Year		Yes	
Area		Yes	
Year × Area			Yes
Fit statistics			
Observations	225,220	225,220	225,220
R^2	0.502	0.709	0.710
Within R^2		0.063	0.063

Clustered (Firm) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Notes: The dependent variable is realized patent value after asinh and a global standardization.

Focusing on the specification with firm and year-by-area fixed effects (Model 3), which absorbs unobserved structural heterogeneity, the upper-tier indicators exhibit significant predictive relationships. Because the dependent variable v_{ijt} is transformed via an inverse hyperbolic sine and standardized, the estimated coefficients seamlessly translate to standard deviation units. Transitioning from a negligible patenting track record to a medium-to-high past patent flow (PAT_{it-1}) shifts the firm's expected area valuation upward by a 0.837 standard deviation relative to the baseline. Similarly, total assets (AT_{it-1}) and R&D stocks (RD_{it-1}) grant significant premiums of 0.109

[7] As discussed in the implementation of the instrument Z_{ijt}, transforming these covariates into non-negative binary scalars allows them to function efficiently as weights $l(\cdot)$.

and 0.072 standard deviations, respectively. This monotonicity confirms that these predetermined covariates belong to the observable state $\mathcal{F}_{ijt}$.

The fit statistics in Table 7 decompose that projection. Model 3 has an R^2 of 0.710, so most of the level of $\check{v}_{ijt}$ is firm history and year-by-area conditions. The within R^2 is 0.063: after those histories, the firm-level stocks still move expected value, with lagged patent flow contributing the largest shift. The residual share of about 0.29 is the combination of event-study noise in the Kogan et al. (2017) valuations, LDA assignment error, and unforecastable market shocks. The moment inequalities use the fitted $\check{v}_{ijt}$, which retains the predictable component and drops that residual. The second-stage critical values treat that fitted valuation as fixed. First-stage sampling error is assumed negligible relative to the inequality bounds, because the projection is estimated on the full panel.

Across the twenty areas, mean absolute deviation of the fitted valuation is clustered around 0.34. Appendix A.7 reports that comparison.

4.2 Reduced-Form Evidence on Firm Entry

Before implementing the structural bounds, it is useful to assess firm entry via standard discrete choice frameworks. Table 8 estimates a reduced-form linear probability model regressing the observed entry indicator d_{ijt} on the projected expected valuation $\check{v}_{ijt}$, competitive congestion $\operatorname{asinh}(N_{jt,-i})$, and the portfolio scale variable $(k_{it} - 1)/2$.

Conditioning on the specification (Model 2) with firm, year, and area fixed effects, valuations matter: higher expected profits increase the likelihood of entry. The coefficient on $\operatorname{asinh}(N_{jt,-i})$ remains significantly positive after expected valuation and portfolio size enter the same regression.

The same OLS specification also generates a statistically strong, positive coefficient on the portfolio measure (0.0995). A firm that occupies k_{it} of twenty areas has mean entry $k_{it}/20$. The regressor is $(k_{it} - 1)/2$, so a coefficient near 0.1 is the adding-up identity of that occupancy rate. Unobserved firm capability can raise both k_{it} and the probability of entering any given area, which is a second reason the linear probability model cannot isolate scope. Distinguishing selection from a true synergy, or from a true coordination cost, requires deviations that hold the unobserved baseline fixed. That is the role of the moment inequalities discussed next.

4.3 Model Estimates

To isolate the marginal incentives that a linear probability model cannot, I partially identify $\theta = (\alpha, \beta, \kappa)$ with the add, drop, and swap inequalities ($\mathbf{M}_1, \mathbf{M}_2, \mathbf{M}_3$) derived in the previous section. Table 9 reports 99 percent confidence sets under the two-step self-normalized procedure of Chernozhukov et al. (2019). That procedure constructs a confidence set for the parameter vector, using a max-type studentized statistic and a two-step critical value that recenters slack moments. The

Table 8: Entry, expected valuation, rival counts, and portfolio size

Dependent Variable:	d	
Model:	(1)	(2)
Variables		
Constant	-0.8239***	
	(0.0334)	
$\tilde{v}_{ijt} = b(\hat{v}_{ijt})$	$7.43 \times 10^{-6**}$	0.0003***
	(3.72×10^{-6})	(3.9×10^{-5})
$\text{asinh}(N_{jt,-i})$	0.1643***	0.0799***
	(0.0063)	(0.0140)
$(k_{it} - 1)/2$	0.0996***	0.0995***
	(0.0002)	(0.0001)
Fixed-effects		
Firm		Yes
Year		Yes
Area		Yes
Fit statistics		
Observations	225,220	225,220
R^2	0.3974	0.4075

Clustered (Firm) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

baseline specification uses the pairwise-interaction portfolio function introduced in Section 3 and add and drop deviations of size two and three. The table shows three rows: no instruments with $|M| = 20$; lagged patent flow with $|M| = 20$; and all three instruments with $|M| = 40$. Appendix C reports the full instrument and subset-size grid.

Table 9: Estimated parameter bounds

Setup	$\lvert M\rvert$	α	β	κ
No IV	20	$[-1.1012, -0.1159]$	$[0.7506, 3.0000]^{\dagger}$	$[0.0010, 24.8606]^{\dagger}$
PAT_{it-1}	20	$[-1.1120, -0.0942]$	$[2.1202, 2.7199]$	$[23.9616, 36.8621]$
RD_{it-1}, AT_{it-1}, PAT_{it-1}	40	$[-1.1159, -0.1834]$	$[2.5093, 2.7125]$	$[24.0649, 36.5644]$

Notes: The table reports 99 percent confidence sets for $\theta = (\alpha, \beta, \kappa)$ under pairwise interactions among entered areas, add and drop deviations of size two and three, and swap moments. The preferred row uses all three instruments and the largest subset sample. Appendix C reports the full instrument and subset-size grid, and the size-two-only grid. A dagger marks a bound that attains an endpoint of the search region. Inference uses the self-normalized two-step procedure of Chernozhukov et al. (2019).

At the specification that uses all three instruments and the largest subset sample, congestion lies in $[-1.1159, -0.1834]$. Scope lies in $[2.5093, 2.7125]$, and the per-area cost lies in $[24.0649, 36.5644]$. Without instruments the inequalities are formed on the pooled sample. Congestion is already strictly negative, but the upper bound on β and the lower bound on κ sit on the edge of the search region, so a per-area cost near zero remains inside that 99 percent set. Lagged patent flow, and then all three firm stocks, enter as non-negative weights that split the sample by predetermined scale and patenting intensity (Canay et al., 2023). Each split is an additional valid inequality. Adding lagged patent flow already pulls β and κ into the interior. At all three instruments and the largest subset sample, the 99 percent set excludes a per-area cost near zero.

The 99 percent sets in Table 9 exclude $\alpha \geq 0$, so they rule out the absence of crowding and any net agglomeration once expected valuations are held fixed. They exclude $\beta \leq 0$, so they rule out a zero or negative scope effect under pairwise interactions among entered areas. Alternative portfolio functions that treat extra areas as diminishing synergies are taken up below.

The inequalities are linear in the observable profit index. There is no residual shock scale to convert into standard deviations. All three parameters are already expressed in units of the first-stage projection of inverse-hyperbolic-sine valuations.

Competitive congestion. The baseline identified set for congestion is $[-1.1159, -0.1834]$. Firms therefore require compensation to enter a more crowded area, holding expected valuation fixed. An incremental unit of inverse-hyperbolic-sine rival counts lowers expected profit by between 0.18 and 1.12 valuation units. Because rival counts enter through the inverse hyperbolic sine, the penalty is steep in thin markets and flattens once an area is saturated: moving from two rivals to three changes the congestion index by about 0.37, while adding the 51st rival to a market of 50 changes it by about 0.02. The reduced-form linear probability coefficient on rival counts is positive, including in the specification that already includes expected valuation and portfolio size. The inequalities put residual congestion in the negative orthant.

Three features of the design isolate that congestion effect. First, the first-stage projection includes firm and year-by-area fixed effects, so common area-time attractiveness is absorbed in $\check{v}_{ijt}$. After that projection, α is disciplined by the residual co-movement between entry and rival counts, holding expected value fixed. Second, a swap compares two areas for the same firm-year and holds portfolio size fixed, so the $\beta g(k)$ term cancels. What remains is the ranking of $\check{v}_j + \alpha h(N_j)$ against $\check{v}_m + \alpha h(N_m)$. Third, the add and drop moments then pin the level of the profit index, bringing κ and β back in. After expected valuations have absorbed firm histories and area-year conditions, α is therefore disciplined by whether firms systematically avoid the more crowded of two areas that look similar in $\check{v}$, and by whether they still enter a crowded area only when $\check{v}$ is high enough to cover κ. As usual for an aggregative congestion term, α is a net effect of business stealing, litigation, and any residual unobserved area attractiveness that the first-stage projection did not absorb. Conditional on expected valuation, crowding dominates.

Portfolio scope. The baseline set for β is $[2.5093, 2.7125]$. Combined with the pairwise-interaction portfolio function, this is a scope benefit: the marginal value of adding one more area equals β times the number of areas already occupied. For a firm already in ten areas, the eleventh area contributes an extra 25.1 to 27.1 valuation units of scope, on top of that area's own expected value, fixed cost, and congestion. The reduced-form portfolio coefficient was also positive. The inequalities isolate it from the capability selection that contaminates the linear probability model. Zero is outside every interval for β in Table 9. Firms do not enter all twenty areas because the per-area fixed cost is large.

Entry cost. The baseline set for κ is $[24.0649, 36.5644]$ in the same units as the first-stage projection. That hurdle is large relative to a typical incremental change in expected valuation or in congestion. Random entry into a low-value, crowded area is therefore unattractive: a firm needs a high expected value, a relatively empty market, or a large realized shock to cover the fixed cost, even though additional areas then generate pairwise synergies. Crowding out, scope benefits, and a steep per-area fixed cost together rationalize both the observed multi-area bundling and the fact that even large electronics firms leave many topics unentered. Those signs remain, at all three instruments and the largest subset sample, when add and drop deviations are restricted to pairs (Table 22 in Appendix C).

The comparison of portfolio functions is itself a finding. Table 10 reports three forms at all three instruments and the largest subset sample.

Table 10: Identified sets under alternative portfolio functions

$g(k)$	α	β	κ
$g(k) = \binom{k}{2}$	$[-1.1159, -0.1834]$	$[2.5093, 2.7125]$	$[24.0649, 36.5644]$
$g(k) = \text{asinh}(k)$	—	—	—
$g(k) = k$	$[-1.0346, -0.9459]$	$[2.1941, 5.0000]^{\dagger}$	$[0.5197, 1.7225]$

Notes: Each row uses all three instruments, the largest subset sample ($|M| = 40$), and the same add, drop, and swap moments as Table 9. The first row is the baseline pairwise-interaction specification. An empty 99 percent confidence set is shown as —. A dagger marks a bound that attains an endpoint of that specification's search region. Appendix C reports the full grid. Inference uses the self-normalized two-step procedure of Chernozhukov et al. (2019).

Under pairwise interactions among entered areas, the 99 percent set for β is nonempty and strictly positive. The concave alternative that applies the inverse hyperbolic sine to portfolio size returns an empty 99 percent confidence set. No combination of congestion, scope, and the per-area cost satisfies the inequalities at the 99 percent critical value under that concave alternative. Diminishing synergies are therefore inconsistent with the observed portfolios under the specification that uses all three instruments and the largest subset sample. A linear portfolio function keeps the same signs, while the interval for β attains the upper edge of the search region and the interval for κ collapses toward 0.5 to 1.7. Appendix C reports the full instrument and subset-size grid for the pairwise-interaction specification, and full-instrument rows for the alternative portfolio functions.

5 Numerical Simulation

To quantify the economic content of the identified sets, I measure how many additional areas firms occupy if the per-area cost falls or if the congestion penalty is scaled toward zero, and I reduce that cost only in a block of semiconductor areas. Appendix D.5 reports a re-ranking that holds portfolio size fixed, hardware subsidies, and the 2010s samples. Appendix D describes how I collect parameter vectors and assign extra areas.

I start from the observed entry pattern and ask which additional areas would raise the observable part of profit. An unoccupied area m is occupied at the current portfolio size k when $\zeta^e_m + \beta \Delta g^+_1(k)$ is positive, where $\zeta^e_m = \check{v}_m + \alpha h(N_m) - \kappa$ and $\Delta g^+_1(k) = g(k+1) - g(k)$ is the extra benefit of occupying one more area, given how many areas the firm already holds. The simulations use the 99 percent confidence set from Table 9 that includes all three instruments and the largest subset sample. The printed intervals for α, β, and κ are profiled: each is the lowest and highest value of that coordinate among vectors that satisfy the criterion. I retain whole vectors $\theta = (\alpha, \beta, \kappa)$ for which the test statistic remains below the critical value jointly, at the same significance level. Each reported range is the minimum and maximum of the predicted object over that collection, which is a smaller set than the product of the three printed intervals. I hold rival counts at their observed values.

5.1 Additional areas after cost cuts

Table 11 reports the average number of extra occupied areas per firm-year and the share of firm-years that occupy at least one extra area, after uniform cuts in the per-area cost.

The 0 percent row keeps the estimated per-area cost unchanged. For each currently unoccupied area, I ask whether occupying it would raise the observable part of profit, given how many areas the firm already holds. Firms gain from occupying more areas at once, so that extra benefit is larger when the firm is already in many areas. Consequently, some empty areas already look worth occupying. Between 13.4 and 16.1 percent of firm-years occupy at least one such extra area, and the average firm-year occupies 0.727 to 0.941 additional areas. The inequalities restrict two- and three-area deviations and allow unobserved shocks. This exercise uses only the observable part of a one-area expansion, so extra entry at the estimated cost can be positive.

I then reduce the per-area cost uniformly. A 25 percent reduction raises the share of firm-years that occupy at least one extra area to between 22.3 and 26.6 percent, and the mean number of extra areas to between 1.522 and 1.961. A 50 percent reduction raises those objects to between 33.2 and 41.3 percent and between 2.725 and 3.758 extra areas. Both objects rise with the size of the cut for every parameter vector in the set.

Table 12 applies the same one-area expansions after scaling the congestion penalty toward zero, holding rival counts fixed.

Table 11: Additional areas occupied after per-area cost cuts

Cost cut (%)	Mean extra areas	Firm-years that expand (%)
0	$[0.727, 0.941]$	$[13.4, 16.1]$
5	$[0.882, 1.062]$	$[15.3, 17.5]$
10	$[1.027, 1.313]$	$[16.9, 20.3]$
15	$[1.178, 1.460]$	$[18.7, 21.8]$
20	$[1.308, 1.803]$	$[20.1, 25.2]$
25	$[1.522, 1.961]$	$[22.3, 26.6]$
30	$[1.696, 2.419]$	$[24.0, 30.8]$
35	$[1.955, 2.640]$	$[26.5, 32.6]$
40	$[2.166, 3.076]$	$[28.4, 36.2]$
45	$[2.492, 3.603]$	$[31.3, 40.2]$
50	$[2.725, 3.758]$	$[33.2, 41.3]$

Notes: Cells are the minimum and maximum over the 99 percent confidence set for $\theta = (\alpha, \beta, \kappa)$ under the specification of Table 9 that uses all three instruments and the largest subset sample, pairwise interactions among entered areas, and add and drop deviations of size two and three. Rival counts are held at their observed values. When a direction is reported, it holds for every vector in that set. Starting from observed entry, I occupy currently unoccupied areas that would raise the observable part of profit after the stated cut in the per-area cost. Mean extra areas is the average number of additional occupied areas per firm-year. Firm-years that expand is the percent of firm-years that occupy at least one extra area. The 0 percent row keeps the estimated per-area cost unchanged.

A 25 percent reduction raises the share of firm-years that occupy at least one extra area to between 14.9 and 17.9 percent. A 50 percent reduction raises that share to between 15.1 and 21.0 percent, and the mean number of extra areas to between 0.860 and 1.390. Setting the congestion penalty to zero raises the share to between 15.8 and 28.2 percent, and the mean number of extra areas to between 0.914 and 2.134. The per-area cost remains large relative to expected patent value, so many empty areas stay unprofitable even when that penalty is set to zero.

5.2 Case Study: Semiconductors

The semiconductor block comprises “Semiconductor Memory Architecture,” “Semiconductor Lithography and Fabrication Processes,” “Integrated Circuit Design, Verification, and Testing,” “Semiconductor Packaging and Interconnect Bonding,” “Transistor Gate Structures and Semiconductor Device Physics,” and “Processor Architecture, Caches, Instruction Execution.” Table 13 reduces the per-area cost in those six areas by 20 or 50 percent in the full sample.

Table 12: Additional areas occupied after a smaller congestion penalty

Congestion cut (%)	Mean extra areas	Firm-years that expand (%)
0	[0.727, 0.941]	[13.4, 16.1]
5	[0.768, 0.969]	[13.9, 16.5]
10	[0.821, 1.006]	[14.6, 16.9]
15	[0.828, 1.022]	[14.7, 17.0]
20	[0.834, 1.070]	[14.8, 17.6]
25	[0.839, 1.098]	[14.9, 17.9]
30	[0.841, 1.134]	[14.9, 18.3]
35	[0.845, 1.164]	[14.9, 18.6]
40	[0.848, 1.214]	[15.0, 19.1]
45	[0.854, 1.285]	[15.1, 19.9]
50	[0.860, 1.390]	[15.1, 21.0]
Set to zero	[0.914, 2.134]	[15.8, 28.2]

Notes: Cells are the minimum and maximum over the 99 percent confidence set for $\theta = (\alpha, \beta, \kappa)$ under the specification of Table 9 that uses all three instruments and the largest subset sample, pairwise interactions among entered areas, and add and drop deviations of size two and three. Rival counts are held at their observed values. When a direction is reported, it holds for every vector in that set. Starting from observed entry, I occupy currently unoccupied areas that would raise the observable part of profit after scaling the congestion penalty toward zero, holding rival counts fixed. A p percent cut multiplies the estimated congestion coefficient by $(1 - p/100)$. The 0 percent row keeps the estimated coefficient. The last row sets the congestion coefficient to zero. Mean extra areas and firm-years that expand are defined as in Table 11.

Table 13: Semiconductor cost subsidy, observed valuations (full sample)

Topic	Label	Observed (%)	Δ at 20% fixed k	Δ at 20% with adds	Δ at 50% fixed k	Δ at 50% with adds
1	Semiconductor Memory Architecture	1.87	$[5.94, 6.54]$	$[5.63, 6.23]$	$[5.96, 6.55]$	$[5.64, 6.24]$
2	Audio and Speech-Signal Processing	4.36	$[-3.12, -2.58]$	$[-3.24, -2.80]$	$[-3.12, -2.58]$	$[-3.24, -2.80]$
3	Semiconductor Lithography and Fabrication Processes	2.48	$[4.65, 4.83]$	$[4.38, 4.52]$	$[4.71, 4.83]$	$[4.40, 4.55]$
4	Integrated Circuit Design, Verification, and Testing	4.53	$[0.79, 2.15]$	$[0.47, 1.85]$	$[0.80, 2.15]$	$[0.47, 1.85]$
5	PCB and Electronic Hardware with Connectors	4.50	$[-3.38, -2.81]$	$[-3.18, -2.81]$	$[-3.38, -2.83]$	$[-3.19, -2.82]$
6	Video Recording, Encoding, and Media Processing	1.98	$[-1.22, -0.82]$	$[-0.97, -0.72]$	$[-1.22, -0.82]$	$[-0.97, -0.72]$
7	Semiconductor Packaging and Interconnect Bonding	2.57	$[4.69, 4.99]$	$[4.38, 4.68]$	$[4.69, 4.99]$	$[4.38, 4.69]$
8	Thin-Film Materials, Deposition and Chemistry	3.51	$[-2.41, -2.18]$	$[-2.17, -1.97]$	$[-2.41, -2.20]$	$[-2.18, -1.99]$
9	Wireless Base-Station and Mobile Communications	2.42	$[-1.63, -1.53]$	$[-1.41, -1.35]$	$[-1.63, -1.54]$	$[-1.41, -1.35]$
10	Clock Generation, Timing Circuits, and Signal Processing	4.06	$[-2.69, -2.28]$	$[-2.76, -2.45]$	$[-2.69, -2.28]$	$[-2.76, -2.45]$
11	Power-Supply and Power-Management Electronics	3.68	$[-2.71, -2.40]$	$[-2.55, -2.36]$	$[-2.71, -2.40]$	$[-2.56, -2.36]$
12	Networking Protocols and Data Infrastructure	2.47	$[-1.48, -1.28]$	$[-1.31, -1.17]$	$[-1.48, -1.31]$	$[-1.31, -1.19]$
13	Transistor Gate Structures and Semiconductor Device Physics	2.03	$[5.66, 6.37]$	$[5.35, 6.05]$	$[5.66, 6.37]$	$[5.36, 6.06]$
14	Imaging Sensors and Camera Pipelines	1.91	$[-1.15, -0.67]$	$[-0.82, -0.48]$	$[-1.15, -0.67]$	$[-0.82, -0.48]$
15	Processor Architecture, Caches, Instruction Execution	3.08	$[2.94, 3.46]$	$[2.67, 3.15]$	$[3.10, 3.46]$	$[2.77, 3.15]$
16	Display Panel Tech (LCD/OLED)	1.02	$[-0.35, 1.18]$	$[0.16, 1.41]$	$[-0.35, 1.18]$	$[0.16, 1.41]$
17	Rotational Electromechanics and Magnetic Actuation	4.39	$[-3.23, -2.90]$	$[-2.99, -2.76]$	$[-3.23, -2.90]$	$[-2.99, -2.76]$
18	Optics, Lasers, and Photonics	2.21	$[-1.37, -1.16]$	$[-1.05, -0.89]$	$[-1.37, -1.17]$	$[-1.05, -0.90]$
19	Analog and Differential Amplifier Circuits	3.57	$[-2.30, -2.05]$	$[-2.22, -2.04]$	$[-2.30, -2.05]$	$[-2.22, -2.04]$
20	Mobile UI and Content Services	3.45	$[-2.45, -2.21]$	$[-2.37, -2.20]$	$[-2.45, -2.21]$	$[-2.37, -2.20]$

Notes: Cells are the minimum and maximum over the 99 percent confidence set for $\theta = (\alpha, \beta, \kappa)$ under the specification of Table 9 that uses all three instruments and the largest subset sample, pairwise interactions among entered areas, and add and drop deviations of size two and three. Rival counts are held at their observed values. The per-area cost in the six semiconductor areas named in the text is reduced by 20 or 50 percent. Fixed k re-ranks the observed portfolio. With adds, I also occupy unentered areas that would raise the observable part of profit. Δ is the change in the area share relative to the observed full-sample share.

At a 20 percent cut, holding each firm-year's number of areas fixed and re-ranking occupied areas, the memory share rises by [5.94, 6.54] percentage points, lithography by [4.65, 4.83], packaging by [4.69, 4.99], device physics by [5.66, 6.37], and processors by [2.94, 3.46]. Integrated circuit design, already a large share, rises by [0.79, 2.15] percentage points. The offset is in unsubsidized areas. "PCB and Electronic Hardware with Connectors" falls by [2.81, 3.38] percentage points, "Rotational Electromechanics and Magnetic Actuation" by [2.90, 3.23], "Audio and Speech-Signal Processing" by [2.58, 3.12], and "Wireless Base-Station and Mobile Communications" by [1.53, 1.63]. The 20 percent and 50 percent columns are close, because a 20 percent cut already places the subsidized areas at the top of the observable ranking.

Rival counts remain at their observed values. If many firms moved into the subsidized areas, congestion there would rise. The targeted cost cuts sit next to recent work on industrial policy toward semiconductors (Juhász et al., 2024; Goldberg et al., 2024). Barwick et al. (2025) recover subsidy effects when rival counts and investment adjust. Bloom et al. (2019) review the broader toolkit of policies that promote innovation. The ranges reported here are partial-equilibrium reallocations at the observed rival counts.

A related exercise reduces the per-area cost only in "Wireless Base-Station and Mobile Communications," "Networking Protocols and Data Infrastructure," and "Mobile UI and Content Services." Appendix D.6 reports that connectivity subsidy. Hardware subsidies and the 2010s samples are in Appendix D.5.

6 Conclusion

This paper develops a data-driven framework to analyze the strategic entry of U.S. electronics firms into multiple technological areas. Departing from the rigid, historically path-dependent administrative taxonomies (such as IPC or CPC codes), I define the innovation areas endogenously. By applying Latent Dirichlet Allocation (LDA) to a vast corpus of unstructured patent texts, I map the continuous, underlying thematic structure of the industry. This approach uncovers the evolutionary stylized facts of the sector: electronics innovation is inherently combinatorial, competitive structures are highly asymmetric, and the industry focus rotated from hardware toward connectivity. To identify the mechanisms driving these observed market configurations, I construct a discrete choice entry model and estimate the parameters by partial identification, using moment inequalities to handle bundling, equilibrium multiplicity, and unobserved firm heterogeneity.

The 99 percent confidence sets support four conclusions. First, firms enter areas where expected patent valuations are higher, which also disciplines the LDA construction: economically meaningless topics would not survive as objects whose valuations rationalize observed portfolios. Second, holding those valuations fixed, additional rivals reduce expected profit. The sets exclude any case in which extra rivals raise expected profit or leave it unchanged. The implied penalty is steep in thin markets and flattens in saturated ones. A reduced-form regression of entry on rival counts is also positive, including in a specification that already includes expected valuation.

Third, firms gain from occupying more areas at once. The sets exclude a zero or negative scope benefit. The reduced-form positive coefficient on portfolio size is consistent with that finding. Under pairwise interactions among entered areas, the 99 percent set on the scope term is nonempty and strictly positive. Treating extra areas as diminishing synergies leaves that set empty when all three instruments and the largest subset sample are used. Fourth, firms still leave many areas unentered because the per-area fixed cost is large. Rival congestion and that fixed cost limit technological expansion. Starting from the observed entry pattern, between 13 and 16 percent of firm-years already have at least one extra area that looks worth occupying at the estimated costs. A 25 percent reduction in the per-area cost raises that share to between 22 and 27 percent, and a 50 percent reduction raises it to between 33 and 41 percent. A 50 percent reduction in how costly rivals are raises that share to between 15 and 21 percent. In a case study of targeted cost reductions, entry rises in "Semiconductor Memory Architecture" and "Semiconductor Lithography and Fabrication Processes." The offset is lower entry in other fields, including "Wireless Base-Station and Mobile Communications."

The paper has three limitations. First, the model is a static game. It abstracts from learning, option value, preemption, and irreversible research outlays. If firms occupy an area in part for later expansion, that option is absorbed in the estimated per-area cost, which is then an upper bound on the contemporaneous hurdle. Second, the sample is listed U.S. electronics firms from 1990 to 2019. The findings and the counterfactuals are a case study of that industry. Startups and privately held innovators are outside the sample, because market-implied patent values require a traded return. Third, the local cost and congestion exercises start from observed portfolios and hold rival counts at their observed values. The reported ranges are partial-equilibrium reallocations at those counts.

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A Data, Methodology, and Stylized Facts

A.1 Latent Dirichlet Allocation (LDA)

Before performing LDA, I first combine the title of the patent and its abstract[8]. Then, I perform preprocessing on the combined text[9]. I proceed to tokenize the combined text, create a set of 'vocabs' consisting of unigrams (a single word) and bigrams (a combination of two words) that will be considered (V), and perform vocab-pruning[10]. This process leaves me with 7,588 vocabs V with other information such as how many times the term appears in the whole corpus (term-frequency) and how many documents have the term (document-frequency)[11]. From here, I construct the document-term matrix of dimension $D \times V$, where D is the number of patents, that will be used as an input for fitting the LDA.

I apply LDA to model electronics patents as mixtures of latent topics, such as semiconductors or wireless communication, each characterized by a distribution over words. Sampling recovers two outputs: a document-topic distribution that assigns topic proportions to each patent, and a topic-word distribution that links words to topics. Besides the document-term matrix, LDA takes three hyperparameters: the number of topics k; the Dirichlet prior α on document-topic proportions, which governs how many topics a patent is allowed to cover; and the Dirichlet prior β on topic-word distributions, which governs how concentrated each topic's vocabulary is.

A Dirichlet prior on the simplex produces sparse mixtures when its parameters are below one and more even mixtures when they exceed one. Each patent d draws topic shares $\theta_d \sim \text{Dirichlet}(\alpha)$. Each topic k draws word probabilities $\phi_k \sim \text{Dirichlet}(\beta)$. Words are then drawn by first sampling a topic and then sampling a word from that topic. Posterior means have the usual Dirichlet-multinomial form: topic shares mix the prior α with token counts, and word probabilities mix the prior β with word counts. I recover those posteriors with the WarpLDA sampler of Chen et al. (2016), which assigns tokens by Metropolis-Hastings. The same Dirichlet-multinomial structure underlies hierarchical, correlated, and dynamic topic models (Teh et al., 2004; Blei and Lafferty, 2006). Those extensions introduce additional hyperparameters that I do not estimate here.

Among the chief challenges of LDA is to identify the optimal hyperparameters, in particular, the number of topics k. One might also argue that different initial priors of α and β can yield different results. I try to mitigate these concerns by computing several measures, summarized in Table 14, that guide the selection of the optimal hyperparameters. The first one is 'perplexity,' which measures how well the model predicts a held-out test set. It is conceptually similar to the inverse likelihood: a lower perplexity score implies that the model generalizes better to unseen data. However, perplexity does not necessarily align with human interpretability, especially in large corpora like patents (see Chang et al. (2009) for detailed discussion on the measurement). For this

[8]The results of my topic modeling are robust if I include title, abstract, and claims, at the cost of higher RAM usage and a significantly longer computation time. Including description is not feasible on my personal machine; that is, the data is about 33.6 GB, and it is difficult to perform topic modeling given the data size and the machine limitation.

[9]First, I translate the combined text into lower case, remove any punctuation and stopwords from the combined text, and I remove all standalone numbers (one or more digits) or letters from the combined text. Finally, I lemmatize the combined text. Example of sentence "Activity 1: I bought 2 coffee makers and the machine is amazing!". After the preprocessing, it becomes "activity buy coffee maker machine amazing".

[10]I set the following restriction: 1. Both unigrams and bigrams must appear at a maximum of 10% of my patent sample. This reduces the redundant words, such as 'invent', being considered in the final set of vocabs V, 2. Both unigrams and bigrams must appear at least 0.1% of my patent sample. The idea of this restriction is to limit overtly technical words, such as 'Fe_3O_4' or 'SiC', being considered in vocabs V.

[11]Using title, abstract, and claims, I have 12,788 vocabs.

measure, I will be looking for an 'elbow', an inflection point where the decrease in perplexity is marginal compared to the increasing number of topics being considered.

Table 14: LDA Evaluation Metrics

Measures	**Purpose**	**Goal**	**Example**
Perplexity	Out-of-sample predictive fit	Lower	"Elbow"
Coherence	Within-topic semantic tightness	Higher	Mean NPMI: $0.2 - 0.3$
Exclusivity	Distinctness between topics	Higher	≥ 0.45
JSD	How much each topic's meaning drifts	Lower	≤ 0.3
Drifted Share	Fraction of topics whose top-10 words change substantially	Lower	max 40% topics drifted

To complement perplexity, I consider coherence, which evaluates how frequently top words within each topic co-occur across the documents. Coherence serves as a proxy for interpretability: higher coherence suggests that the top words in a topic form a semantically meaningful group, increasing confidence that the topic reflects a real innovation cluster (e.g., "chip," "circuit," "transistor" likely point to semiconductors). In particular, I will be using the NPMI-based coherence in evaluating the topic quality (see Mimno et al. (2011) for detailed discussion on the measurement). I optimize the topic model's hyperparameters to achieve a mean NPMI between 0.2 and 0.3, while also ensuring that the minimum NPMI remains around 0.1, to preserve the interpretability of even the weakest topics.

In addition to coherence, I include exclusivity, which assesses how uniquely a word belongs to a single topic, i.e., topic distinctiveness. A high exclusivity score indicates that top words of a topic do not overlap significantly with other topics, thus ensuring distinct innovation areas. While coherence favors commonality among words, exclusivity ensures separation between topics, both of which are desirable but often trade off (see Bischof and Airoldi (2012) for more discussions).

Beyond internal topic quality metrics, I also assess the semantic stability of topics over time. To evaluate this, I split the corpus into three time waves (e.g., 1990-1999, 2000-2009, and 2010–2019), which generally capture the standard narrative of technological evolution in the electronics industry, and compute the Jensen-Shannon divergence (JSD) between the topic-word distributions estimated in each wave. The JSD is a symmetric measure of how dissimilar the vocabulary of each topic is across time periods. In my implementation, JSD can take values in the range of zero and one. A low JSD indicates that topics retain similar meanings over time, which is desirable for maintaining a consistent mapping between topics and innovation areas. In particular, I am looking for the mean JSD below 0.3 when considering the optimal hyperparameters[12].

In addition, I define a measure called drifted share, which calculates the proportion of topics whose top keywords change substantially between time waves. A low drifted share implies that

[12]See for example Lu et al. (2017). Their treatment of the extended JSD measure is applied to compare vast Twitter (now X) corpora across different time periods.

most topics remain stable in meaning, reinforcing their interpretation as persistent innovation domains. For example, there should be a maximum of 40% topics with share drifted more than 30% across period, when evaluating the optimal hyperparameter. Together, JSD and drifted share help identify topic models that are not only coherent and exclusive, but also temporally stable, which is essential for defining innovation areas and for the model implementation used in the next part of this paper[13]. One could also think of this as a way to mitigate the concerns raised in the dynamic, correlated topic model, as discussed by Blei and Lafferty (2006).

To find the optimal hyperparameters, I employ the following procedure (See Algorithm 1 in Appendix A.3, and Appendix A.4 for the fine-tuning result). First, I employ a 5-fold cross validation. That is, for each year, I randomly divide the patents into five parts. I will then fit the LDA for four parts and use the estimated model for the other part when evaluating and optimizing the hyperparameters. I repeat this exercise five times, each with different folds as training and validation sets. During this cross validation, I begin the iterative search[14]. In the first step, I grid search for the number of optimal topics k, while keeping other hyperparameters (α and β) fixed. Based on the evaluation metrics described above, I identify the optimal range for k. Next, I search over values of α, again holding the other parameters fixed, and select its optimal range using the same set of metrics, then obtain the optimal α and its bounds. I then repeat the process for β. This procedure is iterated until further refinements to the hyperparameters do not yield significant improvements in the evaluation metrics.

As discussed earlier, the outputs of the LDA model are the document-topic and topic-word distributions. For each topic, I extract the top 25 most probable words and assign an interpretable label to the topic. To assist with labeling, I utilize two large language models (OpenAI's ChatGPT o3 and xAI's Grok 3 + Think) to propose candidate labels based on the top words associated with each topic. I then iterate between the two models until their suggested labels converge or are manually reconciled, using the agreed-upon label as the final topic label.

A.2 From topic proportions to area weights

The document-topic posterior derived above is the source of the patent-area weights used in Section 3. A document d in the LDA corpus is a patent p filed by firm i in period t. A topic k is an innovation area $j \in \{1, \ldots, J\}$, with $J = 20$ in the fitted model. The Dirichlet-multinomial posterior mean already stated in the generative process is therefore a patent-level distribution over areas,

$$w_{pjt} \;\equiv\; \theta_{pj} \;=\; \frac{N_{pj} + \alpha_j}{\sum_{j'=1}^{J} (N_{pj'} + \alpha_{j'})},$$

where N_{pj} is the number of tokens in patent p assigned to area j, and α_j is the Dirichlet prior on document-topic proportions (the hyperparameter α of this appendix, not the congestion parameter of the entry model). By construction of the Dirichlet,

$$w_{pjt} \geq 0 \qquad \text{and} \qquad \sum_{j=1}^{J} w_{pjt} = 1.$$

[13]I employ two procedures to match the distance matrix: greedy matching and Hungarian matching. Greedy matching does not require a 1-to-1 match between topics in one period to another, while Hungarian matching will enforce a 1-to-1 match. See Wang and McCallum (2006), Niekler and Jähnichen (2012), and Lu et al. (2017) for discussions.

[14]This algorithm is conceptually similar to the Expectation-Maximization (EM) algorithm in statistics, or to coordinate ascent and iterative grid search methods used in machine learning.

A single patent can therefore assign positive weight to several areas at once, with the weights giving the patent's proportionate exposure to each area.

When I move from the patent to the firm-year objects used in estimation, I set any weight below 1 percent to zero, so that a patent does not contribute to areas where its estimated presence is negligible. The remaining fitted weights are left as they are. The area-specific dollar value of the patent is then the product

$$A_{ipjt} \;=\; w_{pjt}\, A_{ipt},$$

where A_{ipt} is the market-implied patent value constructed in Appendix B.1. Summing over the firm's patents in the period gives the realized area valuation that enters the first-stage projection,

$$\sum_{p \in \mathcal{P}_{it}} A_{ipjt} \;=\; \sum_{p \in \mathcal{P}_{it}} w_{pjt}\, A_{ipt}.$$

The same weights define the firm-year entry indicator in Section 3: firm i is recorded as entering area j in period t when the average patent weight in that area is at least 1 percent,

$$d_{ijt} \;\equiv\; \mathbf{1}\left\{ \frac{1}{|\mathcal{P}_{it}|} \sum_{p \in \mathcal{P}_{it}} w_{pjt} \geq \bar{w} \right\}, \qquad \bar{w} = 0.01.$$

A.3 LDA Algorithm

Algorithm 1 Heuristic Hyperparameter Tuning Procedure for LDA

Input: Document-term matrix (DTM), Term-Co-occurence matrix (TCM), initial values for (k, α, β)
Set seed for the random number generator, e.g., 1234
Split data using 5-fold cross validation
Fit LDA on the set of training data
Cross-validate on set of test data
repeat
  Step 1: Tune k ▷ Grid search over k, fix α, β
    Evaluate perplexity, coherence, exclusivity, JSD, drifted share
    Update $k \leftarrow k^*$
  Step 2: Tune α ▷ Grid search over α, fix k, β
    Evaluate same metrics
    Update $\alpha \leftarrow \alpha^*$
  Step 3: Tune β ▷ Grid search over β, fix k, α
    Evaluate same metrics
    Update $\beta \leftarrow \beta^*$
until No significant improvement in evaluation metrics
Output: Optimal (k^*, α^*, β^*)

Important notions:

- Document-term matrix (DTM) will be used as the main input for fitting LDA. Dimension: $D \times V$, where D is the number of document (patent in this case) and V is the number of vocabs.
- Term Co-occurence matrix (TCM) is an important input toc compute one of the evaluation metrics 'coherence' of a topic. Dimension $V \times V$. To construct TCM, we do the following:
  - First, convert DTM into a binary matrix B of shape $D \times V$.
  - Then, TCM is simply $B^T \cdot B$.

A.4 Fine-tuning LDA

As indicated in Algorithm 1, I first start with grid search of the number of topics k. I do this sequence in two parts; first, I sequence from 5 to 100, and qualitatively evaluate the metrics explained earlier in Section 2.2. Then, I sequence from 15 to 25. From here, I settle the number of topics k at 20.

The second step is to perform a grid search for α. First, I search for α in this region, $\{0.01, 0.02, 0.05, 0.1, 0.5, 1\}$. Then, I sequence, with width of 0.01, from 0.01 to 0.15. For this step, I finally chose the hyperparameter α to be at 0.05.

The final step is to perform a grid search for β. I first search for β in this region, (0.001, 0.002, 0.005, 0.007, 0.01, 0.02, 0.05, 0.07, 0.1, 0.2, 0.5, 0.7). Then, I sequence from 0.001 to 0.015 with a width of 0.001. From here, I chose the hyperparameter β to be at 0.001.

To summarize the steps and the resulting optimal hyperparameters chosen in this paper, see Table 15 below.

Table 15: Summary of LDA finetuning

Hyperparameters	**Steps and Results**
Number of topics k	1. Grid search from 5 to 100, with width of 5 2. Grid search from 15 to 25, with width of 1 Optimal number of topics $k^* = 20$
Document-topic priors α	1. Grid search in the region (0.01, 0.02, 0.05, 0.1, 0.5, 1) 2. Grid search from 0.01 to 0.15, with width of 0.01 Optimal $\alpha^* = 0.05$
Topic-word priors β	1. Grid search in the region (0.001, 0.002, 0.005, 0.007, 0.01, 0.02, 0.05, 0.07, 0.1, 0.2, 0.5, 0.7) 2. Grid search from 0.001 to 0.015, with width 0.001 Optimal $\beta^* = 0.001$

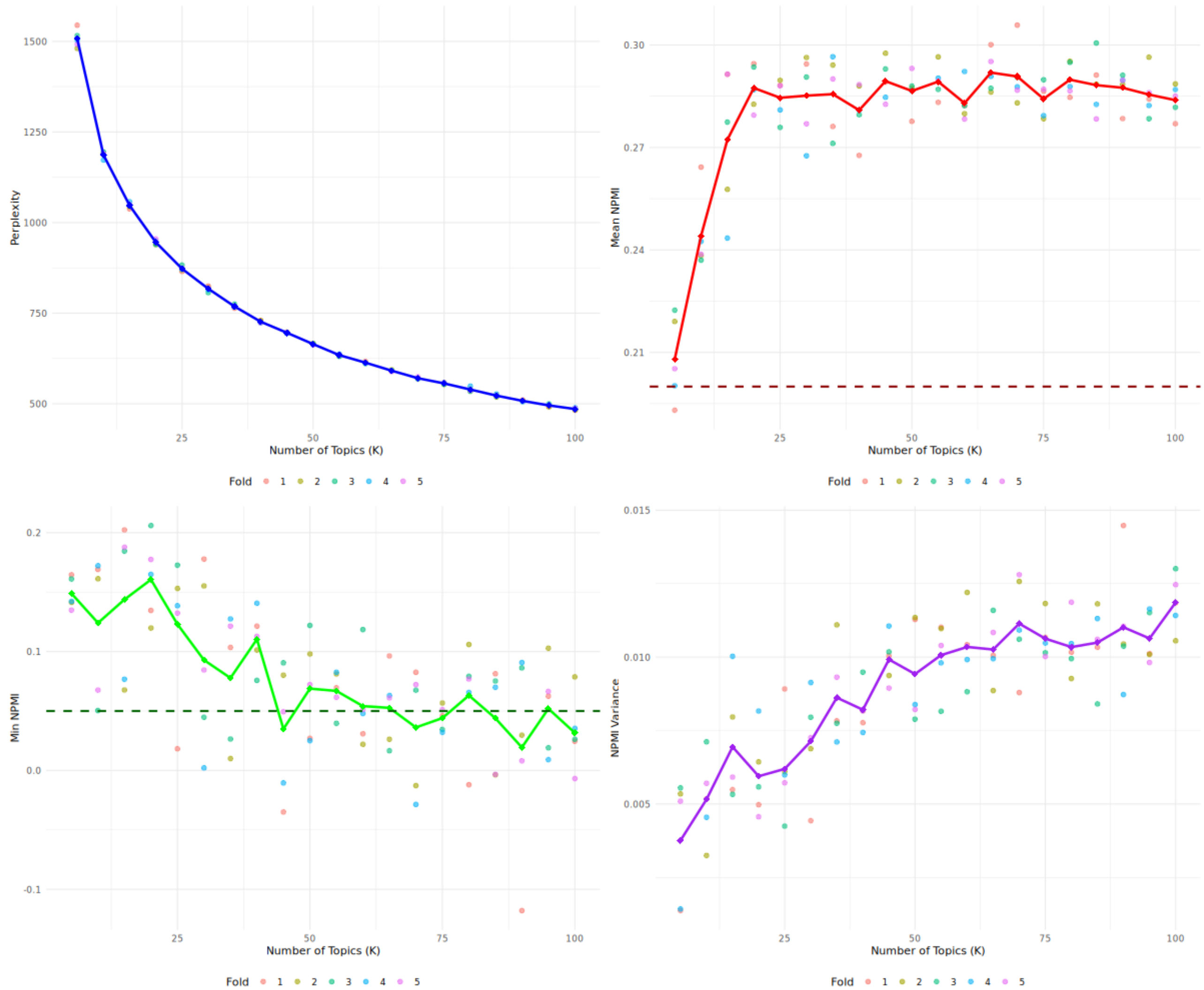
The coarse search over $k \in \{5, \ldots, 100\}$ is the plot that disciplines $k = 20$. Figure 1 shows that the decline in perplexity slows after $k = 20$, while mean NPMI sits in the target band of 0.2 to 0.3 and min NPMI is highest at $k = 20$. Figure 2 shows that mean exclusivity remains above 0.75 in that neighborhood. Figures 3 and 4 show decade-to-decade Jensen-Shannon divergence and drifted share. Mean JSD stays below 0.3 for k between 5 and 20 under Hungarian matching. I therefore set $k = 20$, $\alpha = 0.05$, and $\beta = 0.001$. Finer grids around those values confirm the same choice.

A.5 Topic labels and top words

Table 16 reports the twenty labels and the highest-probability words. The main text lists labels only.

Table 16: Topics and Top Words

Figure 1: Perplexity and coherence. Coarse grid of k from 5 to 100.



Topic	Label	Top Words
1	Semiconductor Memory Architecture	cell, memory_cell, line, row, array, bite_line, column, memory_device, memory_array, flash, word_line, nonvolatile, flash_memory, nonvolatile_memory, word, erase, cell_include, semiconductor_memory, bite, sram, memory_element, refresh, cell_array, transmission_line, dram
2	Audio and Speech-Signal Processing	value, estimate, symbol, calculate, speech, parameter, metric, measure, correlation, sequence, train, coefficient, sample, tone, set, noise, audio_signal, model, threshold, threshold_value, echo, audio, pilot, sound
3	Semiconductor Lithography and Fabrication Processes	dielectric, etch, dielectric_layer, mask, metal, pattern, silicon, metal_layer, deposit, layer_form, photoresist, material, barrier, nitride, expose, dielectric_material, first_dielectric, barrier_layer, mask_layer, oxide_layer, trench, remove, oxide, sacrificial
4	Integrated Circuit Design, Verification, and Testing	test, design, integrate, integrate_circuit, mask, logic, programmable, port, logic_device, ic, programmable_logic, layout, circuitry, circuit_design, output_port, pld, test_mode, method_test, device_test, test_signal, input_port, functional, test_circuit, architecture
5	PCB and Electronic Hardware with Connectors	connector, board, cable, circuit_board, house, module, end, print_circuit, assembly, electronic, conductor, mate, mount, antenna, 2_end, electronic_component, electrical_connector, first_end, socket, contact, print, receptacle, enclosure, component

Topic	Label	Top Words
6	Video Recording, Encoding, and Media Processing	record, video, stream, encode, reproduce, code, record_medium, block, picture, video_signal, decode, medium, frame, video_datum, reproduction, datum_stream, information_record, encoder, motion_vector, bit, compress, datum_record, datum_block, parity
7	Semiconductor Packaging and Interconnect Bonding	die, package, pad, bond, chip, conductive, solder, bump, first_surface, bond_pad, semiconductor_die, lead, semiconductor_package, semiconductor_chip, 2_surface, mems, microelectronic, ball, lead_frame, package_include, electrically, mold, interposer, leadframe
8	Thin-Film Materials, Deposition and Chemistry	composition, glass, temperature, particle, polymer, resin, gas, compound, film, coat, fuel, polish, ceramic, acid, mixture, reaction, electrolyte, carbon, catalyst, fuel_cell, melt, solution, hydrogen, powder
9	Wireless Base-Station and Mobile Communications	station, wireless, base_station, wireless_communication, transmission, ue, channel, radio, communication_system, uplink, user_equipment, downlink, mobile_station, equipment, resource, control_channel, equipment_ue, subframe, radio_communication, report, reception, wireless_device, communication_device
10	Clock Generation, Timing Circuits, and Signal Processing	clock, frequency, phase, clock_signal, delay, filter, analog, digital, oscillator, rf, output_signal, loop, input_signal, pulse, digital_signal, datum_signal, rf_signal, mixer, adc, pll, signal_2, signal_generate, gain, baseband
11	Power-Supply and Power-Management Electronics	power, power_supply, mode, switch, consumption, power_amplifier, supply, battery, power_source, low_power, transformer, ac, dc, power_consumption, power_converter, power_control, reduce_power, power_management, output_power, wireless_power, first_power, 2_power, primary, secondary
12	Networking Protocols and Data Infrastructure	network, packet, protocol, node, message, mode, link, network_node, communication_network, datum_packet, ip, service, traffic, network_device, access_point, subscriber, gateway, access_network, session, first_network, wireless_network, network_element, area_network, entity
13	Transistor Gate Structures and Semiconductor Device Physics	region, gate, fin, gate_electrode, drain, dope, sourcedrain, gate_structure, region_2, semiconductor_layer, conductivity_type, drain_region, source_drain, channel_region, region_form, conductivity, implant, region_first, epitaxial, sourcedrain_region, active_region, dopant, ptype, transistor
14	Imaging Sensors and Camera Pipelines	image, sensor, display, camera, image_datum, object, screen, information_process, image_process, capture, image_sensor, image_signal, process_apparatus, section, image_display, touch, color, display_unit, scene, capture_image, apparatus_image, image_capture, first_image, display_screen
15	Processor Architecture, Caches, Instruction Execution	processor, instruction, address, cache, bus, register, execution, command, store, host, memory_controller, interrupt, cpu, execute, memory_access, buff, thread, graphic, storage_device, controller, pipeline, entry, operand, transaction
16	Display Panel Tech (LCD/OLED)	electrode, liquid_crystal, panel, crystal, pixel, display_device, liquid, display, display_panel, first_electrode, 2_electrode, pixel_electrode, crystal_display, solidstate, organic, cpu, gate_line, organic_light, film_transistor, photoelectric, solidstate_image, electrode_2, lcd, plasma_display
17	Rotational Electromechanics and Magnetic Actuation	member, air, magnetic, head, shaft, rotate, roller, main_body, refrigerator, magnet, rotation, actuator, wash, motor, rotor, door, nozzle, cartridge, drum, inner, body, compressor, movable, laundry
18	Optics, Lasers, and Photonics	optical, light, lens, laser, fiber, beam, wavelength, light_source, optical_fiber, emit, laser_beam, waveguide, reflect, light_beam, optical_signal, refractive, incident, polarization, reflection, light_guide, illumination, grate, reflector, optical_system
19	Analog and Differential Amplifier Circuits	voltage, current, bias, differential, transistor, amplifier, resistor, reference_voltage, stage, current_source, input_terminal, output_terminal, voltage_level, first_voltage, supply_voltage, charge_pump, 2_voltage, terminal, output_node, driver, output_voltage, bias_voltage, voltage_first, voltage_2
20	Mobile UI and Content Services	user, key, content, server, user_interface, mobile_device, application, authentication, telephone, compute_device, item, computer, encrypt, mobile, security, client, encryption, location, database, computer_program, remote, phone, game, file

A.6 Competitor counts and valuations by decade

Tables 18 and 19 report rival counts and patent values by topic and decade. The main text reports the corresponding share tables.

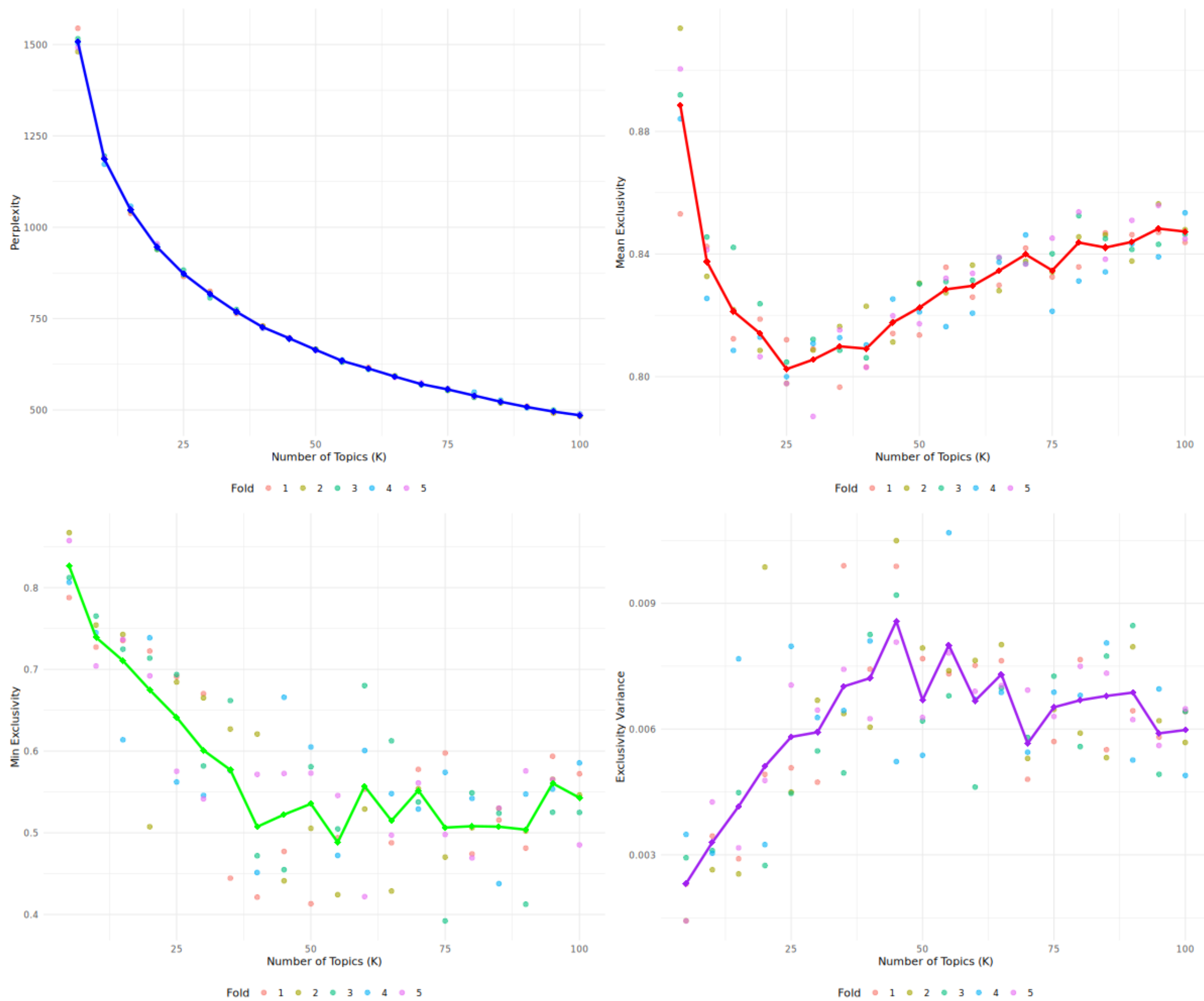
Table 18: Competitor counts by topic and decade

Topic	Label	1990s	2000s	2010s	Mean	SD
1	Semiconductor Memory Architecture	78.21	85.87	56.23	75.59	18.39
2	Audio and Speech-Signal Processing	144.49	182.38	135.87	156.65	31.26
3	Semiconductor Lithography and Fabrication Processes	90.13	110.33	81.42	95.58	19.25
4	Integrated Circuit Design, Verification, and Testing	155.05	194.85	134.22	164.87	35.76
5	PCB and Electronic Hardware with Connectors	142.59	164.38	127.97	147.16	23.77
6	Video Recording, Encoding, and Media Processing	76.85	94.41	63.95	80.25	17.79
7	Semiconductor Packaging and Interconnect Bonding	77.84	116.57	98.62	97.7	21.15
8	Thin-Film Materials, Deposition and Chemistry	128.76	130.93	89.09	119.62	24.89
9	Wireless Base-Station and Mobile Communications	75.97	110.2	89.96	92.42	20.77
10	Clock Generation, Timing Circuits, and Signal Processing	142.86	173.42	122.23	149.24	31.58
11	Power-Supply and Power-Management Electronics	123.27	155.58	119.92	134.65	25.09
12	Networking Protocols and Data Infrastructure	75.11	110.63	85	91.02	23.24
13	Transistor Gate Structures and Semiconductor Device Physics	74.05	95.88	71.04	81.55	16.07
14	Imaging Sensors and Camera Pipelines	64.76	83.04	80.26	75.56	13.67
15	Processor Architecture, Caches, Instruction Execution	106.24	139.03	105.6	118.48	25.26
16	Display Panel Tech (LCD/OLED)	38.63	45.8	36.6	40.83	8.18
17	Rotational Electromechanics and Magnetic Actuation	150.07	147.76	113.25	139.95	22.69
18	Optics, Lasers, and Photonics	78.65	96.04	74.21	84.11	14.26
19	Analog and Differential Amplifier Circuits	129.29	155.28	112.9	135.01	27.31
20	Mobile UI and Content Services	113.88	145.6	109.77	124.84	23.82

Table 19: Patent valuation by topic and decade

Topic	Label	1990s	2000s	2010s	Mean	SD
1	Semiconductor Memory Architecture	21.14	13.25	32.22	20.94	185.68
2	Audio and Speech-Signal Processing	59.98	67.45	164.38	89	946.12
3	Semiconductor Lithography and Fabrication Processes	37.22	27.85	57.59	38.79	334.05
4	Integrated Circuit Design, Verification, and Testing	48.72	44.95	99.48	60.03	511.19
5	PCB and Electronic Hardware with Connectors	33.69	36.43	116.4	55.48	702.7
6	Video Recording, Encoding, and Media Processing	21.46	19.92	43.85	26.5	263.56
7	Semiconductor Packaging and Interconnect Bonding	25.43	23.09	65.97	34.72	294.2
8	Thin-Film Materials, Deposition and Chemistry	57.21	53.77	63.61	57.52	563.75
9	Wireless Base-Station and Mobile Communications	32.71	37.62	159.88	66.48	1091.89
10	Clock Generation, Timing Circuits, and Signal Processing	56.24	46.29	85.24	59.75	421.75
11	Power-Supply and Power-Management Electronics	22.07	24.39	67.12	34.25	305.21
12	Networking Protocols and Data Infrastructure	32.29	39.73	90.41	49.69	499.55
13	Transistor Gate Structures and Semiconductor Device Physics	26.28	20.12	57.12	31.69	264.17
14	Imaging Sensors and Camera Pipelines	14.15	22.22	115.92	42.74	793.96
15	Processor Architecture, Caches, Instruction Execution	94.46	63.72	197.53	108.7	1345.49
16	Display Panel Tech (LCD/OLED)	4.44	6.15	27.71	10.93	185.31
17	Rotational Electromechanics and Magnetic Actuation	27.87	24.15	62.5	35.16	311.01
18	Optics, Lasers, and Photonics	17.74	18.64	39.9	23.64	265.96
19	Analog and Differential Amplifier Circuits	57.8	45.97	83	59.65	439.15
20	Mobile UI and Content Services	45.76	81.45	281.62	118.45	2037.15

Figure 2: Perplexity and exclusivity. Coarse grid of k from 5 to 100.



A.7 First-stage fit by area

Table 20 reports, for each of the twenty areas, the standard deviation of realized v_{ijt} and of fitted $\check{v}_{ijt}$, together with mean absolute deviation. Fitted values shrink noise, and the fit is similar across topics.

Figure 3: Jensen-Shannon divergence and drifted share, 1990s to 2000s. Coarse grid of k from 5 to 100.

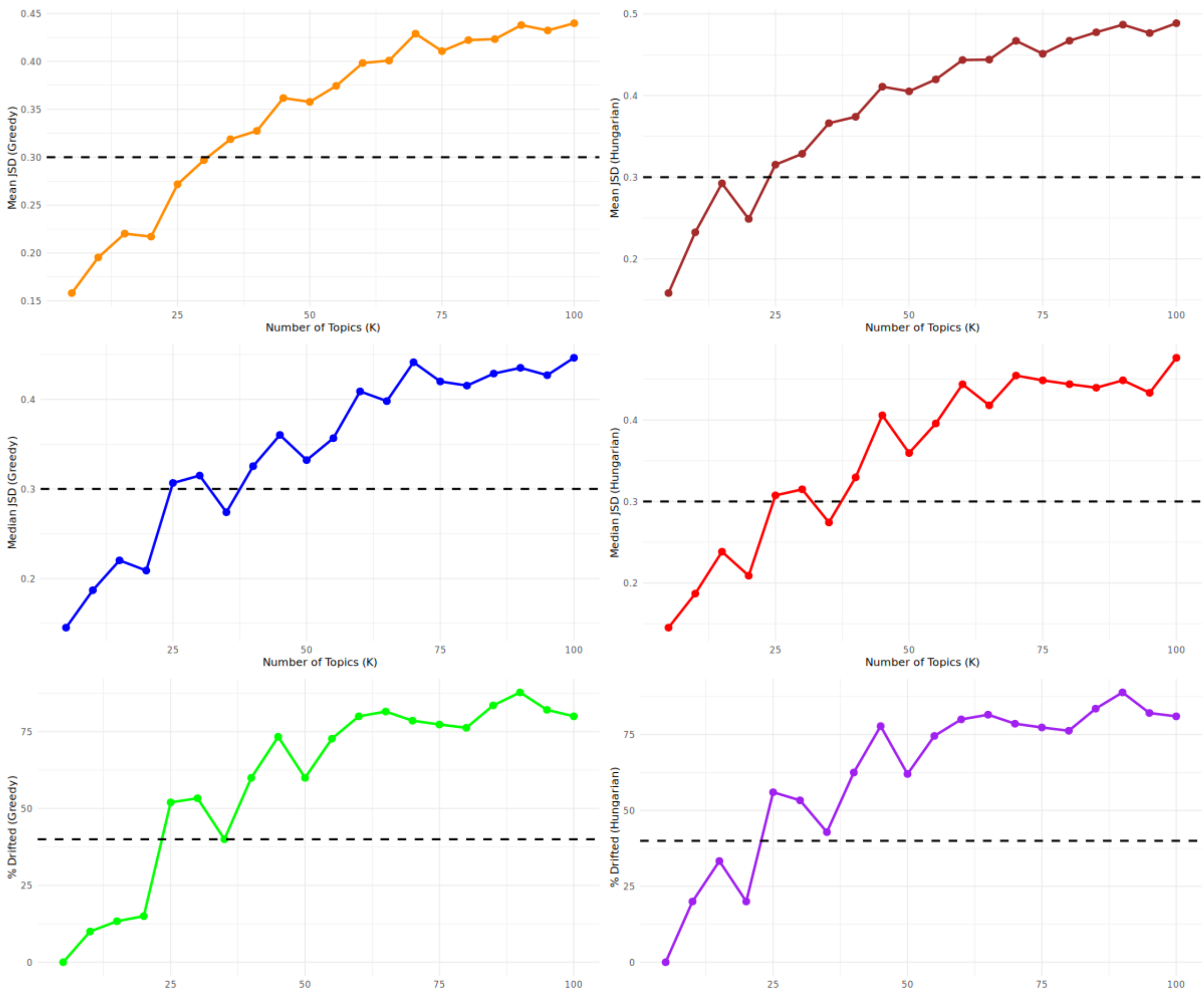


Figure 4: Jensen-Shannon divergence and drifted share, 2000s to 2010s. Coarse grid of k from 5 to 100.

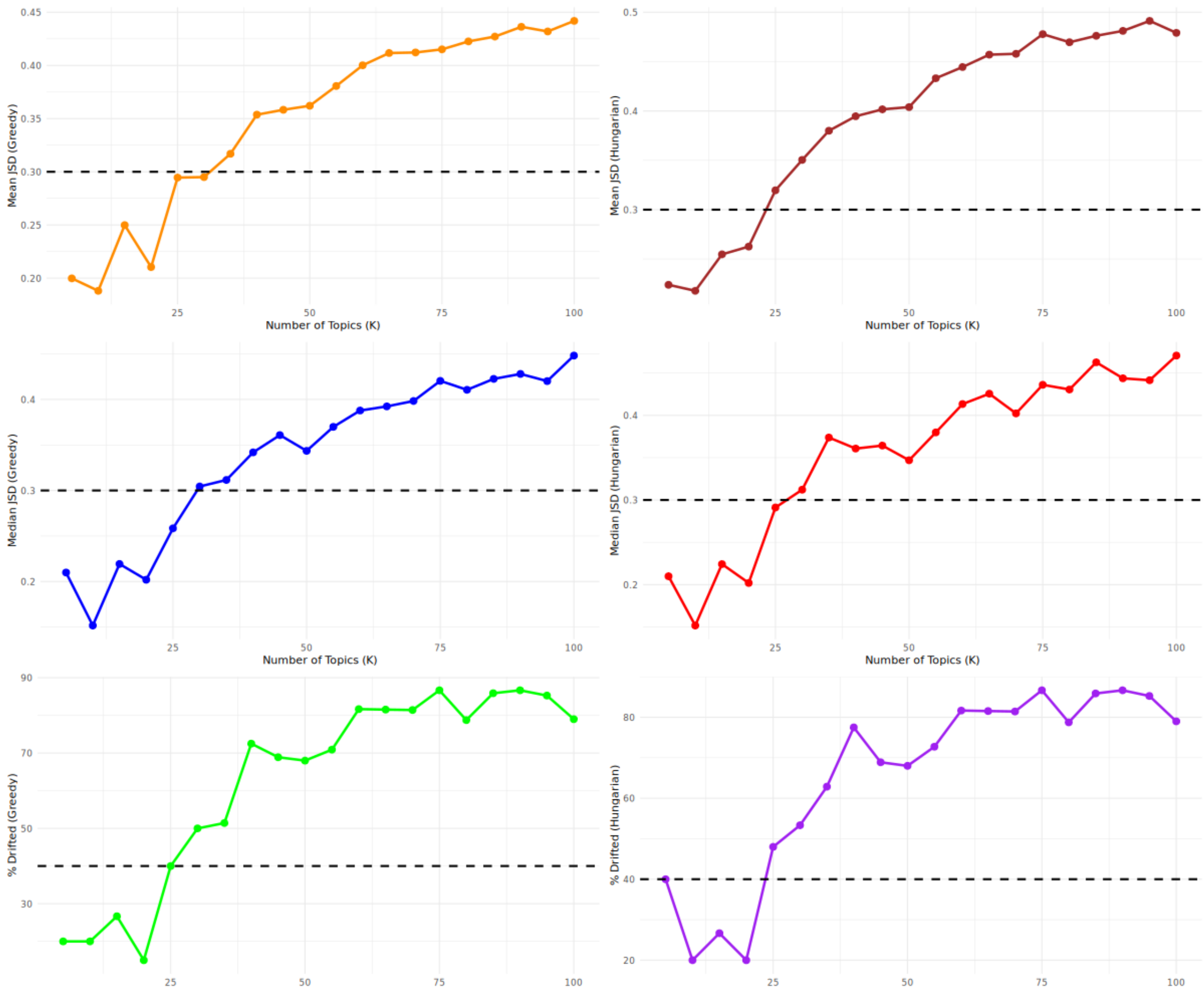


Table 20: Comparison of Predictive Performance Across Specifications

Topic	Label	sd(v_{ijt})	sd($\hat{v}^{fe}_{ijt}$)	sd($\hat{v}^{pool}_{ijt}$)	MAD^{fe}	MAD^{pool}
1	Semiconductor Memory Architecture	0.8525	0.8356	0.8279	0.2795	0.2757
2	Audio and Speech-Signal Processing	1.1342	0.8356	0.8442	0.3828	0.3820
3	Semiconductor Lithography and Fabrication Processes	0.9727	0.8356	0.8342	0.3160	0.3156
4	Integrated Circuit Design, Verification, and Testing	1.1036	0.8356	0.8396	0.3661	0.3663
5	PCB and Electronic Hardware with Connectors	1.0636	0.8356	0.8416	0.4055	0.4048
6	Video Recording, Encoding, and Media Processing	0.8756	0.8356	0.8283	0.2832	0.2788
7	Semiconductor Packaging and Interconnect Bonding	0.9823	0.8356	0.8425	0.3080	0.3070
8	Thin-Film Materials, Deposition and Chemistry	1.0148	0.8356	0.8290	0.3691	0.3729
9	Wireless Base-Station and Mobile Communications	0.9366	0.8356	0.8390	0.2875	0.2884
10	Clock Generation, Timing Circuits, and Signal Processing	1.1375	0.8356	0.8389	0.4072	0.4072
11	Power-Supply and Power-Management Electronics	1.0299	0.8356	0.8417	0.3325	0.3354
12	Networking Protocols and Data Infrastructure	0.9842	0.8356	0.8365	0.3221	0.3231
13	Transistor Gate Structures and Semiconductor Device Physics	0.9542	0.8356	0.8343	0.3013	0.2995
14	Imaging Sensors and Camera Pipelines	0.8659	0.8356	0.8366	0.2827	0.2807
15	Processor Architecture, Caches, Instruction Execution	1.0744	0.8356	0.8365	0.3245	0.3254
16	Display Panel Tech (LCD/OLED)	0.6922	0.8356	0.8274	0.3289	0.3229
17	Rotational Electromechanics and Magnetic Actuation	1.0386	0.8356	0.8342	0.4109	0.4102
18	Optics, Lasers, and Photonics	0.8809	0.8356	0.8339	0.3230	0.3215
19	Analog and Differential Amplifier Circuits	1.1271	0.8356	0.8386	0.3987	0.4002
20	Mobile UI and Content Services	1.0503	0.8356	0.8428	0.3337	0.3350
	Mean	0.9886	0.8356	0.8364	0.3382	0.3376

Notes: v_{ijt} is realized patent value after asinh and a global standardization (the same left-hand side as Table 7). $\hat{v}^{fe}_{ijt}$ and $\hat{v}^{pool}_{ijt}$ are fitted values from Table 7 Model 2 (firm, year, and area effects) and Model 3 (firm and year-by-area effects), respectively. MAD = Mean Absolute Deviation between actual and fitted values on this common scale.

B Patent valuation and moment inequalities

This appendix records three objects that Section 3 uses without derivation. Appendix B.1 restates the market-implied patent values of Kogan et al. (2017) in the notation of this paper and aggregates them to the firm-year-area level. Appendix B.2 records the characterization of the optimal bundle and then derives the three families of linear moment inequalities. Appendix B.3 records subset sampling, the test statistic, and the static-game caveats.

B.1 Market-implied patent values

Each patent p filed by firm i and granted in period t is assigned a dollar value A_{ipt} from the filtered stock-market reaction constructed by Kogan et al. (2017). I take their filtered series as given and only change notation so that the object lines up with the area-level valuation v_{ijt} in Section 3. What follows is their measurement model, written for a patent of firm i rather than for a generic grant j.

The USPTO publishes grants on a known announcement day. Absent other news, the change in the firm's market value on that day would recover only the unanticipated portion of the patent's private value. If ϖ_{ipt} is the market's ex-ante probability that the application is granted and ξ_{ipt} is the present value of the associated monopoly rents, then

$$\Delta V_{ipt} \;=\; \big(1-\varpi_{ipt}\big)\,\xi_{ipt}. \tag{4}$$

The announcement therefore understates the full private value by the factor $1-\varpi_{ipt}$. The object is the unexpected increment associated with the grant, together with that scaling. The full present value of the underlying research program is a different quantity. LDA weights allocate that grant-level increment across innovation areas. Both steps introduce measurement error, which is why Section 4 projects realized valuations onto the firm's predetermined state. Following Kogan et al. (2017), I work with a three-day window $[t, t+2]$ around the grant date. Even inside that window the idiosyncratic return still mixes the patent with unrelated news. Write the market-adjusted return of firm i around the grant of patent p as

$$R_{ipt} \;=\; \nu_{ipt} + \varsigma_{ipt}, \tag{5}$$

where ν_{ipt} is the value of the patent as a fraction of the firm's market capitalization and ς_{ipt} is the residual return. Kogan et al. (2017) treat ν_{ipt} as a truncated normal, $\nu_{ipt} \sim N^{+}(0, \sigma^2_{\nu,it})$, and the residual as Gaussian, $\varsigma_{ipt} \sim N(0, \sigma^2_{\varsigma,it})$, independent of ν_{ipt}. The filtered fraction is then

$$\mathbb{E}[\nu_{ipt} \mid R_{ipt}] \;=\; \delta_{it}\,R_{ipt} + \sqrt{\delta_{it}}\,\sigma_{\varsigma,it}\,\frac{\varphi\big(-\sqrt{\delta_{it}}\,R_{ipt}/\sigma_{\varsigma,it}\big)}{1-\Phi\big(-\sqrt{\delta_{it}}\,R_{ipt}/\sigma_{\varsigma,it}\big)}, \tag{6}$$

where φ and Φ are the standard normal density and distribution and

$$\delta_{it} \;=\; \frac{\sigma^2_{\nu,it}}{\sigma^2_{\nu,it}+\sigma^2_{\varsigma,it}} \tag{7}$$

is the signal-to-noise ratio. Let M_{it} be the firm's market capitalization on the day before the announcement, and let N_{ipt} be the number of patents granted to the firm on the same day. The dollar value that I take from their filtered series is

$$A_{ipt} \;\equiv\; \xi_{ipt} \;=\; \big(1-\bar{\varpi}\big)^{-1} \frac{1}{N_{ipt}}\, \mathbb{E}[\nu_{ipt} \mid R_{ipt}]\, M_{it}. \tag{8}$$

The factor $(1-\bar{\varpi})^{-1}$ undoes the average grant probability in (4). The factor $1/N_{ipt}$ splits a day's filtered value equally across contemporaneous grants. I use their published filtered dollar series, in both nominal and 1982 dollars. The grant-day filter and the signal-to-noise ratio are those of Kogan et al. (2017).

The remaining step is the one that is specific to this paper. The grant-day filter delivers a single dollar value per patent. The entry model is defined on innovation areas. Using the LDA weights w_{pjt} of Appendix A.2, I allocate the patent across areas by

$$A_{ipjt} \;=\; w_{pjt}\, A_{ipt}. \tag{9}$$

Summing over the firm's patents in the period produces the realized area total

$$\sum_{p\in\mathcal{P}_{it}} A_{ipjt} \;=\; \sum_{p\in\mathcal{P}_{it}} w_{pjt}\, A_{ipt}. \tag{10}$$

Section 3 maps this total through a monotone transformation $b(\cdot)$ and a mean-zero measurement error,

$$v_{ijt} \;=\; b\Bigg(\sum_{p\in\mathcal{P}_{it}} w_{pjt}\, A_{ipt}\Bigg) + e_{ijt}. \tag{11}$$

In the empirical implementation, $b(\cdot)$ is the inverse hyperbolic sine. The structural expected valuation $\check{v}_{ijt}$ is the fitted value from projecting that transformed total on the firm's observable state. The residual e_{ijt} absorbs filter noise in A_{ipt}, sampling noise in the LDA weights, and any remaining aggregation error. Firms do not observe e_{ijt} at the time of entry, so $\mathbb{E}[e_{ijt} \mid \mathcal{F}_{ijt}] = 0$.

B.2 Derivation of the moment inequalities

I first take Result 1 to Result 2. I then derive Results 3–5 from those two results. Throughout, rivals' equilibrium counts $N_{jt,-i}$ are taken as given. The set of areas is finite. The unobservable ϵ_{ijt} is a composite ex-post forecast error with $\mathbb{E}[\epsilon_{ijt} \mid \mathcal{F}_{ijt}] = 0$. Instruments Z are a subset of $\mathcal{F}_{ijt}$, and $l(\cdot)$ maps them to a nonnegative scalar. All objects that appear in the deterministic indexes below are therefore measurable with respect to $\mathcal{F}_{ijt}$.

Write $S = \{j : d_{ijt} = 1\}$ for the observed bundle, $k_{it} = |S|$, and

$$\zeta^{e}_{ijt} \;=\; \check{v}_{ijt} + \alpha\, h(N_{jt,-i}) - \kappa$$

for the expected profitability index of area j. Result 1 gives

$$\mathbb{E}[\Pi_{it} \mid \mathcal{F}_{ijt}] \;=\; \sum_{j\in S} \zeta^{e}_{ijt} + \beta\, g(k_{it}). \tag{12}$$

Nash equilibrium requires that no unilateral deviation raise expected profit. For any alternative bundle S',

$$\mathbb{E}\big[\Pi_{it}(S') - \Pi_{it}(S) \;\big|\; \mathcal{F}_{ijt}\big] \;\leq\; 0. \tag{13}$$

Characterization of the optimal bundle. The portfolio function g depends on the entered set only through how many areas it contains. Holding rivals' counts fixed, the firm can therefore rank areas by a single index and then choose how many of the highest-ranked areas to enter. I record that argument as three lemmas. They use only Result 1 and the restriction on g. The number of areas J is finite.

The first lemma is the swap calculation. The second says that, at a fixed size, the best bundles are those that never leave a better area out while keeping a worse one in. The third says that the firm then only has to compare those best-of-size values across the $J+1$ possible sizes.

Lemma 1 (Swap identity) *Let S be a bundle of size k, and let $j \in S$ and $m \notin S$. The swapped bundle $S' = (S \setminus \{j\}) \cup \{m\}$ has the same size, and*

$$\mathbb{E}\big[\Pi_{it}(S') - \Pi_{it}(S) \bigm| \mathcal{F}_{ijt}\big] \;=\; \zeta^{e}_{imt} - \zeta^{e}_{ijt}. \tag{14}$$

Proof. By (12), expected profit is the sum of the indexes of the entered areas plus βg of the number of entered areas. The swap drops j and adds m, so the two sums differ only by $\zeta^{e}_{imt} - \zeta^{e}_{ijt}$. The count is still k, so $\beta g(k)$ is the same on both bundles and cancels. ■

Lemma 2 (Fixed portfolio size) *Fix a size $k \in \{0, \dots, J\}$. A bundle S of size k maximizes expected profit among all bundles of size k if and only if it satisfies the threshold property*

$$\min_{j \in S} \zeta^{e}_{ijt} \;\geq\; \max_{m \notin S} \zeta^{e}_{imt}, \tag{15}$$

where the minimum over the empty set is $+\infty$ and the maximum over the empty set is $-\infty$. Every such bundle attains

$$U^{e}_{k} \;=\; \sum_{r=1}^{k} \zeta^{e}_{(r)} + \beta\, g(k), \tag{16}$$

with $\zeta^{e}_{(1)} \geq \cdots \geq \zeta^{e}_{(J)}$ the decreasing rearrangement of the indexes. That k-set is unique if and only if $\zeta^{e}_{(k)} > \zeta^{e}_{(k+1)}$, reading $\zeta^{e}_{(0)} = +\infty$ and $\zeta^{e}_{(J+1)} = -\infty$.

Proof. The empty bundle ($k = 0$) and the full set of areas ($k = J$) satisfy (15) automatically: there is either nothing inside to compare, or nothing outside. I therefore treat a generic size k.

First, suppose S violates (15). Then the firm has entered some area j whose index is strictly below the index of some area m that it left out. Lemma 1 says that swapping m for j raises expected profit while keeping the same size. So a size-k maximizer cannot violate the threshold property.

Second, suppose S satisfies (15), and let T be any other bundle of the same size. The two sets differ by equally many areas: the extras in S and the extras in T can be lined up one for one. Every extra in S is an entered area, so by (15) its index is at least as large as the index of every area left out of S. Every extra in T was left out of S. Pairing them therefore gives

$$\sum_{j \in S \setminus T} \zeta^{e}_{ijt} \;\geq\; \sum_{m \in T \setminus S} \zeta^{e}_{imt}. \tag{17}$$

The areas that S and T share cancel. The portfolio term $\beta g(k)$ is the same on both bundles. Hence expected profit on S is at least as large as expected profit on T. Every threshold k-set is therefore optimal for that size.

Third, any threshold k-set must include every area whose index is strictly above the k-th largest index $\zeta^e_{(k)}$, and then enough areas tied at that cutoff to reach size k. All of those sets add the same k largest indexes, so they all deliver the same expected profit U^e_k in (16). If $\zeta^e_{(k)} > \zeta^e_{(k+1)}$, the cutoff is a single area and that set is unique. If several areas share the cutoff, any selection among those tied areas that fills out size k is optimal for that k. ■

Lemma 3 (Optimal size) *The $J+1$ numbers $U^e_0, \ldots, U^e_J$ attain a maximum. Write k^* for any maximizer. A bundle S maximizes expected profit over the power set if and only if $|S| = k^*$ for some such k^* and S satisfies* (15). *In particular, every optimal bundle is a set of k^* areas with the largest indexes.*

Proof. Every bundle has some size k between 0 and J. Lemma 2 says that, within size k, the best expected profit is the single number U^e_k, and it is attained exactly on the threshold k-sets. The firm then compares the $J+1$ numbers $U^e_0, \ldots, U^e_J$. A finite list of real numbers has a largest value, so at least one best size k^* exists. The best expected profit in the whole problem is therefore $\max_k U^e_k$, and the bundles that attain it are exactly the threshold sets of those best sizes. ■

Lemmas 1–3 are Result 2. Several sizes can share that largest value of U^e_k. In that case any of those sizes is an optimal k^*.

Adding M areas. Fix an exogenous subset $\mathcal{M}$ of size M. Write $D^0_{\mathcal{M},it} \equiv \prod_{j\in\mathcal{M}}(1-d_{ijt})$ for the indicator that the realized bundle entered none of these areas. On the event $\{D^0_{\mathcal{M},it} = 1\}$, one has $\mathcal{M} \cap S = \emptyset$, so the deviation bundle $S' = S \cup \mathcal{M}$ is well defined and has portfolio size $k_{it} + M$. Equation (12) yields

$$\mathbb{E}[\Pi_{it}(S') \mid \mathcal{F}_{ijt}] = \sum_{j\in S} \zeta^e_{ijt} + \sum_{j\in\mathcal{M}} \zeta^e_{ijt} + \beta\, g(k_{it}+M),$$
$$\mathbb{E}[\Pi_{it}(S) \mid \mathcal{F}_{ijt}] = \sum_{j\in S} \zeta^e_{ijt} + \beta\, g(k_{it}).$$

The expected profit difference is therefore

$$\mathbb{E}[\Delta\Pi_{it} \mid \mathcal{F}_{ijt}] \;=\; \sum_{j\in\mathcal{M}} \zeta^e_{ijt} + \beta\big(g(k_{it}+M) - g(k_{it})\big) \;=\; \eta^+_{i,\mathcal{M},t}, \tag{18}$$

where $\Delta g^+_M(k_{it}) \equiv g(k_{it}+M) - g(k_{it})$ and

$$\eta^+_{i,\mathcal{M},t} \;=\; \sum_{j\in\mathcal{M}} \big(\check{v}_{ijt} + \alpha\, h(N_{jt,-i}) - \kappa\big) + \beta\, \Delta g^+_M(k_{it}).$$

Revealed preference (13) on the event $\{D^0_{\mathcal{M},it} = 1\}$ implies $\eta^+_{i,\mathcal{M},t} \le 0$. Off that event the product $D^0_{\mathcal{M},it}\, \eta^+_{i,\mathcal{M},t}$ is zero. Hence

$$D^0_{\mathcal{M},it}\, \eta^+_{i,\mathcal{M},t} \;\le\; 0 \tag{19}$$

almost surely. Because $l(Z_{i,\mathcal{M},t}) \ge 0$, the same product remains nonpositive after multiplication by the instrument. Taking expectations delivers the unconditional moment

$$\mathbf{M}_1(Z_{i,\mathcal{M},t};\theta) \;=\; \mathbb{E}\big[D^0_{\mathcal{M},it}\, \eta^+_{i,\mathcal{M},t}\, l(Z_{i,\mathcal{M},t})\big] \;\le\; 0, \tag{20}$$

which is Result 3.

Dropping M **areas.** For the same exogenous $\mathcal{M}$, write $D^1_{\mathcal{M},it} \equiv \prod_{j\in\mathcal{M}} d_{ijt}$ for the indicator that every area in the subset was entered. On the event $\{D^1_{\mathcal{M},it} = 1\}$, one has $\mathcal{M} \subset S$ and therefore $k_{it} \geq M$. The deviation bundle is $S' = S \setminus \mathcal{M}$, with portfolio size $k_{it} - M$. The same expansion of (12) gives

$$\mathbb{E}[\Delta\Pi_{it} \mid \mathcal{F}_{ijt}] \;=\; -\sum_{j\in\mathcal{M}} \zeta^e_{ijt} - \beta\big(g(k_{it}) - g(k_{it} - M)\big) \;=\; -\eta^-_{i,\mathcal{M},t}, \tag{21}$$

where $\Delta g^-_M(k_{it}) \equiv g(k_{it}) - g(k_{it} - M)$ and

$$\eta^-_{i,\mathcal{M},t} \;=\; \sum_{j\in\mathcal{M}} \big(\check{v}_{ijt} + \alpha\, h(N_{jt,-i}) - \kappa\big) + \beta\, \Delta g^-_M(k_{it}).$$

Revealed preference requires $-\eta^-_{i,\mathcal{M},t} \leq 0$ on the event $\{D^1_{\mathcal{M},it} = 1\}$. Therefore

$$D^1_{\mathcal{M},it}\,\big(-\eta^-_{i,\mathcal{M},t}\big) \;\leq\; 0 \tag{22}$$

almost surely. Multiplying by $l(Z_{i,\mathcal{M},t}) \geq 0$ and taking expectations yields

$$\mathbf{M}_2(Z_{i,\mathcal{M},t};\theta) \;=\; \mathbb{E}\big[D^1_{\mathcal{M},it}\,\big(-\eta^-_{i,\mathcal{M},t}\big)\, l(Z_{i,\mathcal{M},t})\big] \;\leq\; 0, \tag{23}$$

which is Result 4.

Pairwise swaps. Let (j,m) be an ordered pair with $j \in S$ and $m \notin S$, so that $D^{\text{swap}}_{j,m,it} \equiv d_{ijt}(1 - d_{imt}) = 1$. The deviation replaces j with m. Portfolio size is unchanged, and therefore $\Delta g = 0$. The expected profit difference collapses to the difference of indexes,

$$\begin{aligned}
\mathbb{E}[\Delta\Pi_{it} \mid \mathcal{F}_{ijt}] &= \zeta^e_{imt} - \zeta^e_{ijt} \\
&= \big(\check{v}_{imt} + \alpha\, h(N_{mt,-i}) - \kappa\big) - \big(\check{v}_{ijt} + \alpha\, h(N_{jt,-i}) - \kappa\big) \\
&= \big(\check{v}_{imt} + \alpha\, h(N_{mt,-i})\big) - \big(\check{v}_{ijt} + \alpha\, h(N_{jt,-i})\big) \;=\; \mu_{ijmt}(\theta).
\end{aligned} \tag{24}$$

The per-area cost κ cancels. Revealed preference requires $\mu_{ijmt}(\theta) \leq 0$ on the event $\{D^{\text{swap}}_{j,m,it} = 1\}$, so

$$D^{\text{swap}}_{j,m,it}\, \mu_{ijmt}(\theta) \;\leq\; 0 \tag{25}$$

almost surely. The unconditional moment is

$$\mathbf{M}_3(Z_{ijmt};\theta) \;=\; \mathbb{E}\big[D^{\text{swap}}_{j,m,it}\, \mu_{ijmt}(\theta)\, l(Z_{ijmt})\big] \;\leq\; 0, \tag{26}$$

which is Result 5. Because the portfolio term drops out, $\mathbf{M}_3$ isolates the congestion parameter α from the scale of $g(\cdot)$.

From pathwise inequalities to unconditional moments. The three products (19), (22), and (25) are already nonpositive as random variables. Passing to an unconditional moment uses only that a nonnegative instrument cannot flip that sign.

Lemma 4 (Nonnegative measurable weights) *Let X be integrable and $\mathcal{F}_{ijt}$-measurable, with $X \leq 0$ almost surely. Let W be nonnegative and $\mathcal{F}_{ijt}$-measurable, and suppose WX is integrable. Then $WX \leq 0$ almost surely, and*

$$\mathbb{E}[WX] \;\leq\; 0. \tag{27}$$

Proof. The event $\{W \geq 0\} \cap \{X \leq 0\}$ has probability one, and on that event the product WX cannot be strictly positive. Hence $WX \leq 0$ almost surely. For an integrable random variable Y,

$$\mathbb{E}[Y] \;=\; \mathbb{E}[Y\mathbf{1}_{\{Y\leq 0\}}] + \mathbb{E}[Y\mathbf{1}_{\{Y>0\}}].$$

The second term vanishes when $P(Y > 0) = 0$. The first term is the expectation of a nonpositive integrable function, so it is at most zero. Taking $Y = WX$ gives (27). ■

Because X is $\mathcal{F}_{ijt}$-measurable, $\mathbb{E}[X \mid \mathcal{F}_{ijt}] = X$, and the tower property records the same object as

$$\mathbb{E}[WX] \;=\; \mathbb{E}\big[W\,\mathbb{E}[X \mid \mathcal{F}_{ijt}]\big]. \tag{28}$$

The right-hand side is the usual instrumenting identity that starts from a conditional moment $\mathbb{E}[X \mid \mathcal{F}_{ijt}] \leq 0$. Here the complete-information assumption already gives the pathwise bound $X \leq 0$, so Lemma 4 applies directly.

I apply the lemma three times, each time with $W = l(Z)$ for a nonnegative function of instruments contained in $\mathcal{F}_{ijt}$.

For adding M areas, set $X = D^0_{\mathcal{M},it}\,\eta^+_{i,\mathcal{M},t}$. The indicator $D^0_{\mathcal{M},it}$ is a function of the observed bundle. Under the information structure of Section 3 the firm has no private type, so that bundle is a function of $\mathcal{F}_{ijt}$ once rivals' actions are taken as given. The index $\eta^+_{i,\mathcal{M},t}$ is a function of $\check{v}$, N, k_{it}, and θ. The first three objects lie in $\mathcal{F}_{ijt}$ or are conditioned on; θ is constant. Hence X is $\mathcal{F}_{ijt}$-measurable. Equation (19) is the pathwise bound $X \leq 0$. Lemma 4 applied to this pair (W, X) is Result 3.

For dropping M areas, set $X = D^1_{\mathcal{M},it}\,(-\eta^-_{i,\mathcal{M},t})$. The indicator is a function of the observed bundle. The index $\eta^-_{i,\mathcal{M},t}$ is a function of $\check{v}$, N, k_{it}, and θ, the same objects as in the add case. Hence X is $\mathcal{F}_{ijt}$-measurable. Equation (22) is the pathwise bound. Lemma 4 applied to this pair is Result 4.

For a pairwise swap, set $X = D^{\text{swap}}_{j,m,it}\,\mu_{ijmt}(\theta)$. The indicator is a function of (d_{ijt}, d_{imt}), and $\mu_{ijmt}(\theta)$ is a function of the two area indexes. Equation (25) is the pathwise bound. Lemma 4 applied to this pair is Result 5.

The only properties required of $l(\cdot)$ for these inequalities to be valid are nonnegativity, $\mathcal{F}_{ijt}$-measurability, and integrability of the product $l(Z)X$. In the reported estimates, $l(Z)$ is a collection of binary firm-year indicators that partition the sample by lagged R&D, assets, and patent flow. A bounded instrument makes $l(Z)X$ integrable whenever X is.

The baseline specification evaluates the inequalities on exogenous pairs and triplets ($M \in \{2, 3\}$) together with the swap pairs, as described in Section 3. The algebra above does not depend on the particular functional form of $g(\cdot)$. The estimates in Section 4 use $g(k) = \binom{k}{2}$.

B.3 Empirical implementation

This subsection records the search over a bounded parameter region, the profiled bounds, and the remaining static-game caveats. Section 3 states how subsets are chosen from $\check{v}_{ijt}$ and how the 99 percent sets are constructed.

Subset sampling. Evaluating every pair, triplet, and ordered swap among $J = 20$ areas is unnecessary for the inequalities. I therefore rank subsets from predicted values $\check{v}_{ijt}$ before looking at realized d_{ijt}. The inequalities keep the leading $|M|$ subsets of that ranking, with $|M| \in \{10, 20, 40\}$

on the instrument grid in Appendix C. I restrict joint deviations to $M \in \{2,3\}$. I omit single-area add and drop moments because, given observed k_{it}, $\Delta g_1^+ = k_{it}$ and $\Delta g_1^- = k_{it} - 1$ are nearly collinear with κ. Pair and triplet deviations break that collinearity. For drop moments I take the subsets with the highest average predicted value. For add moments I take the subsets with the lowest average predicted value. For swaps I rank the 380 ordered pairs by divergence in $\check{v}$ and draw from the top, the bottom, and the middle of that ranking. The specification that uses all three instruments and the largest subset sample keeps $|M| = 40$ per deviation type.

Test statistic and profiling. The object is a confidence set for θ, not a point estimate. The test statistic $T(\theta)$ is the max-type studentized sample moment of Chernozhukov et al. (2019). The critical value $c(1-a,\theta)$ is their two-step self-normalized procedure at $a = 0.01$. I search a bounded parameter region and keep the best draws as starting values. If the test statistic still exceeds the critical value, I minimize that slack without constraints to obtain a feasible start. Profiled bounds for each coordinate then solve

$$\min_{\theta} y'\theta \qquad \text{and} \qquad \max_{\theta} y'\theta \qquad \text{subject to} \qquad T(\theta) \le c(1-a,\theta).$$

Static game. The static game answers how a firm composes its contemporaneous portfolio and how rival counts in the same period shift expected profit. It leaves out learning, option value, preemption, and irreversible research outlays. If firms enter in part for option value, estimated κ absorbs that option and is an upper bound on the purely contemporaneous fixed cost. If entry is sticky, α mixes contemporaneous crowding with the shadow cost of a persistent rival stock. Firm and year-by-area effects absorb a time-invariant capability level and common area-time conditions. A sequential-move or dynamic estimator would require an equilibrium-selection rule, a transition for rival counts, and a continuation value. I do not impose those objects.

C Robustness of the identified sets

The baseline confidence sets in Section 4 use the pairwise-interaction portfolio function and add and drop deviations of size two and three. Table 21 reports the full instrument and subset-size grid. Table 22 repeats that grid after dropping size-three add and drop moments. Table 10 in the main text reports a three-row comparison of portfolio functions. This appendix reports that pairwise-interaction grid and, for the alternative portfolio functions, the remaining full-instrument rows.

Table 21: Estimated Parameter Bounds under Different Instrument Setups

Setup	$\|M\|$	α	β	κ
No IV	20	$[-1.1012, -0.1159]$	$[0.7506, 3.0000]^{\dagger}$	$[0.0010, 24.8606]^{\dagger}$
RD_{it-1}	20	$[-1.1102, -0.2103]$	$[0.5497, 3.0000]^{\dagger}$	$[0.0010, 33.6446]^{\dagger}$
AT_{it-1}	20	$[-1.1079, -0.1490]$	$[1.4106, 3.0000]^{\dagger}$	$[0.9579, 34.0712]$
PAT_{it-1}	20	$[-1.1120, -0.0942]$	$[2.1202, 2.7199]$	$[23.9616, 36.8621]$
RD_{it-1}, AT_{it-1}	20	$[-1.1094, -0.2035]$	$[0.3003, 3.0000]^{\dagger}$	$[3.6530, 33.5233]$
RD_{it-1}, PAT_{it-1}	20	$[-1.1130, -0.1978]$	$[2.0413, 2.7512]$	$[23.8255, 36.2676]$
AT_{it-1}, PAT_{it-1}	20	$[-1.1128, -0.1351]$	$[2.1317, 2.7518]$	$[24.2249, 25.8959]$
RD_{it-1}, AT_{it-1}, PAT_{it-1}	10	$[-1.1116, 0.1284]^{*}$	$[1.8246, 3.0000]^{\dagger}$	$[29.8345, 39.8271]$
RD_{it-1}, AT_{it-1}, PAT_{it-1}	20	$[-1.1137, -0.1933]$	$[1.9685, 2.7165]$	$[23.7423, 36.3865]$
RD_{it-1}, AT_{it-1}, PAT_{it-1}	40	$[-1.1159, -0.1834]$	$[2.5093, 2.7125]$	$[24.0649, 36.5644]$

Notes: The table reports 99 percent confidence sets for $\theta = (\alpha, \beta, \kappa)$ under pairwise interactions among entered areas, add and drop deviations of size two and three, and swap moments. I search α in $[-5.000, 5.0]$, β in $[-5.0, 3.0]$, and κ in $[0.001, 60.0]$. A dagger marks a bound that attains an endpoint of that region. An asterisk marks an interval that contains zero. An empty 99 percent confidence set is shown as —. Inference uses the self-normalized two-step procedure of Chernozhukov et al. (2019). The reported intervals are 99 percent confidence sets (significance level $a = 0.01$).

This appendix also reports alternative portfolio functions at all three instruments.

C.1 Alternative portfolio functions

Under the pairwise-interaction technology, a positive β means that pairwise interactions scale with the number of unique pairs among entered areas, so the marginal value of the next area rises with portfolio size. Table 23 reports the remaining full-instrument rows, including subset samples other than $|M| = 40$ and the convex form that multiplies portfolio size by its inverse hyperbolic sine.

Three facts stand out.

First, applying the inverse hyperbolic sine to portfolio size, the concave form that would make a positive β a diminishing synergy, returns an empty 99 percent confidence set at all three instruments for both subset samples in Table 23. The inequalities reject that concave technology inside the region over which I search.

Second, the linear technology that treats each extra area as a constant shift keeps the same signs ($\alpha < 0$, $\beta > 0$) but leaves β stacked against its numerical upper bound. The per-area scope bonus is then weakly identified, and κ collapses to a small interval near 0.5 to 1.7. Linear scope is hard

Table 22: Estimated Parameter Bounds, Add/Drop $M = 2$ Only

Setup	$\lvert M \rvert$	α	β	κ
No IV	20	$[-1.1012, -0.2181]$	$[-0.1186, 3.0000]^{\dagger *}$	$[0.0010, 33.1450]^{\dagger}$
RD_{it-1}	20	$[-1.1066, -0.2150]$	$[0.8406, 3.0000]^{\dagger}$	$[4.1742, 33.0397]$
AT_{it-1}	20	$[-1.1061, -0.1537]$	$[0.5941, 3.0000]^{\dagger}$	$[1.0490, 35.1227]$
PAT_{it-1}	20	$[-1.1103, -0.0984]$	$[2.5093, 2.7037]$	$[24.4153, 36.7952]$
RD_{it-1}, AT_{it-1}	20	$[-1.1045, -0.2072]$	$[1.3898, 3.0000]^{\dagger}$	$[3.5934, 33.5299]$
RD_{it-1}, PAT_{it-1}	20	$[-1.1113, -0.2046]$	$[2.1008, 2.7115]$	$[24.2436, 36.1781]$
AT_{it-1}, PAT_{it-1}	20	$[-1.1111, -0.1422]$	$[2.0434, 2.7281]$	$[23.0879, 36.6169]$
RD_{it-1}, AT_{it-1}, PAT_{it-1}	10	$[-1.1090, 0.1240]^{*}$	$[1.6051, 3.0000]^{\dagger}$	$[23.3359, 39.7499]$
RD_{it-1}, AT_{it-1}, PAT_{it-1}	20	$[-1.1118, -0.2000]$	$[2.1583, 2.7525]$	$[24.2009, 36.3744]$
RD_{it-1}, AT_{it-1}, PAT_{it-1}	40	$[-1.1139, -0.1910]$	$[2.2430, 2.8001]$	$[24.4035, 36.4880]$

Notes: The table uses the same pairwise-interaction portfolio function and the same instrument and subset-size grid as Table 21, with add and drop deviations of size two only. I search α in $[-5.000, 5.0]$, β in $[-5.0, 3.0]$, and κ in $[0.001, 60.0]$. A dagger marks a bound that attains an endpoint of that region. An asterisk marks an interval that contains zero. An empty 99 percent confidence set is shown as —. Inference uses the self-normalized two-step procedure of Chernozhukov et al. (2019). The reported intervals are 99 percent confidence sets (significance level $a = 0.01$).

Table 23: Sensitivity of identified sets to the portfolio function $g(k)$

$g(k)$	Moments	$\lvert M \rvert$	α	β	κ
$g(k) = \binom{k}{2}$	Add/Drop $M \in \{2, 3\}$	10	$[-1.1116, 0.1284]^{*}$	$[1.8246, 3.0000]^{\dagger}$	$[29.8345, 39.8271]$
$g(k) = \binom{k}{2}$	Add/Drop $M \in \{2, 3\}$	20	$[-1.1137, -0.1933]$	$[1.9685, 2.7165]$	$[23.7423, 36.3865]$
$g(k) = \binom{k}{2}$	Add/Drop $M \in \{2, 3\}$	40	$[-1.1159, -0.1834]$	$[2.5093, 2.7125]$	$[24.0649, 36.5644]$
$g(k) = \binom{k}{2}$	Add/Drop $M = 2$	10	$[-1.1090, 0.1240]^{*}$	$[1.6051, 3.0000]^{\dagger}$	$[23.3359, 39.7499]$
$g(k) = \binom{k}{2}$	Add/Drop $M = 2$	20	$[-1.1118, -0.2000]$	$[2.1583, 2.7525]$	$[24.2009, 36.3744]$
$g(k) = \binom{k}{2}$	Add/Drop $M = 2$	40	$[-1.1139, -0.1910]$	$[2.2430, 2.8001]$	$[24.4035, 36.4880]$
$g(k) = k$	Add/Drop $M \in \{2, 3\}$	20	$[-1.0099, -0.8550]$	$[3.9179, 5.0000]^{\dagger}$	$[0.5434, 1.6890]$
$g(k) = k$	Add/Drop $M \in \{2, 3\}$	40	$[-1.0346, -0.9459]$	$[2.1941, 5.0000]^{\dagger}$	$[0.5197, 1.7225]$
$g(k) = k \operatorname{asinh}(k)$	Add/Drop $M \in \{2, 3\}$	20	$[-0.9838, -0.1931]$	$[5.7567, 6.6328]$	$[16.1648, 34.0053]$
$g(k) = k \operatorname{asinh}(k)$	Add/Drop $M \in \{2, 3\}$	40	$[-0.9800, -0.3389]$	$[5.7983, 6.7156]$	$[16.9979, 22.4178]$
$g(k) = \operatorname{asinh}(k)$	Add/Drop $M \in \{2, 3\}$	20	—	—	—
$g(k) = \operatorname{asinh}(k)$	Add/Drop $M \in \{2, 3\}$	40	—	—	—

Notes: Each row uses all three instruments: lagged R&D, assets, and patent flow. The baseline specification uses pairwise interactions among entered areas and add and drop deviations of size two and three. The inverse-hyperbolic-sine portfolio function returns an empty confidence set at these instrument configurations. The searched region differs by portfolio function. A dagger marks a bound that attains an endpoint of that specification's region. An asterisk marks an interval that contains zero. An empty 99 percent confidence set is shown as —. Inference uses the self-normalized two-step procedure of Chernozhukov et al. (2019). The reported intervals are 99 percent confidence sets (significance level $a = 0.01$).

to separate from the fixed cost.

Third, multiplying portfolio size by its inverse hyperbolic sine is another convex form. It remains feasible, keeps $\alpha < 0$ and $\beta > 0$, and produces a tighter interval for κ than the pairwise-interaction technology at the largest subset sample. I keep the pairwise-interaction function as the baseline because it is the technology written in Section 3, and because its interval for β in the baseline specification is interior to the region over which I search.

Taken together, at all three instruments and the largest subset sample the feasible forms preserve crowding out and a positive scope term. Under a concave portfolio function the identified set empties.

D Construction of the numerical simulations

This appendix records how I compute the objects in Section 5. The economic questions remain in the main text.

D.1 Parameter vectors

The simulations use the 99 percent confidence set from Table 9 that includes all three instruments and the largest subset sample. The printed intervals for α, β, and κ are profiled: each is the lowest and highest value of that coordinate among vectors that satisfy the criterion. I retain whole vectors $\theta = (\alpha, \beta, \kappa)$ for which $T(\theta) \leq c(1 - a, \theta)$ holds jointly, at the same significance level. Each reported range is the minimum and maximum of the predicted object over that collection, which is a smaller set than the product of the three printed intervals.

D.2 Assigning extra areas

The cost-cut and congestion-cut tables start from the observed entry pattern. I leave currently occupied areas occupied. Among empty areas I select the one with the largest $\zeta_m^e = \check{v}_m + \alpha h(N_m) - \kappa_m$, where κ_m equals the common per-area cost except when a targeted cut applies to a subset of areas. I occupy that area when $\zeta_m^e + \beta \Delta g_1^+(k)$ is positive at the current size k, raise k by one, and repeat while the increment remains positive. When two areas share the same index, I occupy the one with the smaller topic number.

A uniform cut of p percent replaces κ with $(1 - p/100)\kappa$. A congestion cut of p percent replaces α with $(1 - p/100)\alpha$. Setting the congestion penalty to zero sets $\alpha = 0$. The 0 percent cost row can still record extra entry, because this assignment uses only the observable part of a one-area expansion.

D.3 Composition at a fixed size

When I hold the number of areas fixed, I occupy the k areas with the largest ζ_m^e. That ranking can replace the observed mix. A common per-area cost does not affect which areas are occupied at a fixed size. The composition table uses 1990s firm-years and 1990s rival counts, and takes the area-year component of the first-stage projection from the 2010s. The 2010s subsidy tables use 2010s firm-years, 2010s valuations, 2010s rival counts, and 2010s portfolios. After that re-ranking I also report the mix obtained by then adding extra areas as above, starting from the re-ranked bundle.

D.4 Rival counts and shocks

Rival counts remain at their observed values. I do not redraw unobserved shocks. Every extra-entry exercise begins at the observed bundle. Every fixed-size exercise begins at the observed number of areas.

D.5 Additional simulations

The main text reports uniform cost cuts, congestion cuts, and the semiconductor case in the full sample. This subsection collects the remaining exercises.

Composition at observed portfolio size. Table 24 holds each 1990s firm-year's number of areas fixed and assigns the areas with the highest observable index under 2010s valuations.

Table 24: 1990s area shares at observed portfolio size under 2010s valuations

Topic	Label	1990s observed (%)	1990s with 2010s valuations (%)	2010s observed (%)
1	Semiconductor Memory Architecture	1.90	[0.90, 1.50]	1.78
2	Audio and Speech-Signal Processing	3.91	[1.48, 2.76]	4.76
3	Semiconductor Lithography and Fabrication Processes	2.27	[1.40, 1.52]	2.67
4	Integrated Circuit Design, Verification, and Testing	4.16	[1.09, 2.19]	4.64
5	PCB and Electronic Hardware with Connectors	4.44	[1.69, 3.43]	4.64
6	Video Recording, Encoding, and Media Processing	1.84	[1.17, 2.30]	2.08
7	Semiconductor Packaging and Interconnect Bonding	1.97	[3.47, 4.12]	3.23
8	Thin-Film Materials, Deposition and Chemistry	3.73	[1.16, 1.83]	3.35
9	Wireless Base-Station and Mobile Communications	1.84	[5.36, 5.93]	3.07
10	Clock Generation, Timing Circuits, and Signal Processing	3.81	[1.66, 2.54]	4.20
11	Power-Supply and Power-Management Electronics	3.30	[1.60, 2.90]	4.08
12	Networking Protocols and Data Infrastructure	1.91	[3.95, 4.60]	2.91
13	Transistor Gate Structures and Semiconductor Device Physics	1.72	[1.51, 2.88]	2.26
14	Imaging Sensors and Camera Pipelines	1.57	[4.98, 6.58]	2.61
15	Processor Architecture, Caches, Instruction Execution	2.60	[1.55, 2.06]	3.54
16	Display Panel Tech (LCD/OLED)	0.91	[2.89, 8.00]	1.15
17	Rotational Electromechanics and Magnetic Actuation	4.82	[1.81, 2.99]	4.29
18	Optics, Lasers, and Photonics	2.05	[1.96, 2.96]	2.48
19	Analog and Differential Amplifier Circuits	3.33	[1.63, 2.39]	3.78
20	Mobile UI and Content Services	2.99	[1.55, 3.51]	3.93
Matched share of the observed 1990s–2010s reallocation: $[-2.58, -1.21]$.				

Notes: Cells are the minimum and maximum over the 99 percent confidence set for $\theta = (\alpha, \beta, \kappa)$ under the specification of Table 9 that uses all three instruments and the largest subset sample, pairwise interactions among entered areas, and add and drop deviations of size two and three. Rival counts are held at their observed values. When a direction is reported, it holds for every vector in that set. Each firm-year keeps its observed number of areas and occupies the areas with the highest observable index under 2010s area-year valuations. The matched share is $1 - \sum_j |\hat{s}_j - s_j^{2010s}| / \sum_j |s_j^{1990s} - s_j^{2010s}|$, reported as a range over the joint set.

The matched share of the observed 1990s–2010s reallocation is $[-2.58, -1.21]$. Values below zero mean that the predicted mix is farther from the 2010s mix than the 1990s mix is.

Hardware subsidies. Table 25 reduces the per-area cost in "PCB and Electronic Hardware with Connectors," "Thin-Film Materials, Deposition and Chemistry," "Display Panel Tech (LCD/OLED)," and "Rotational Electromechanics and Magnetic Actuation" in the full sample. Table 26 repeats the exercise in the 2010s sample.

Semiconductor subsidies in the 2010s. Table 27 applies the semiconductor cost cut in the 2010s sample.

D.6 Connectivity subsidy

Table 28 reports a connectivity subsidy related to Section 5. The main text reports the semiconductor subsidy in the full sample. Hardware subsidies and the 2010s samples are above. Here I reduce the per-area cost by 20 or 50 percent in "Wireless Base-Station and Mobile Communications," "Networking Protocols and Data Infrastructure," and "Mobile UI and Content Services." I then recompute area shares both at each firm-year's observed number of areas and after adding currently unoccupied areas whose observable increment is positive. At these subsidies, the 20 percent and 50 percent columns are nearly the same because a 20 percent cut already re-ranks the three subsidized areas to the top of the observable index, so a deeper cut barely changes shares.

Table 25: Hardware cost subsidy, observed valuations (full sample)

Topic	Label	Observed (%)	Δ at 20% (fixed k)	Δ at 20% (with adds)	Δ at 50% (fixed k)	Δ at 50% (with adds)
1	Semiconductor Memory Architecture	1.87	$[-0.66, 0.34]$	$[-0.34, 0.51]$	$[-0.66, 0.34]$	$[-0.34, 0.51]$
2	Audio and Speech-Signal Processing	4.36	$[-2.80, -1.90]$	$[-2.97, -2.16]$	$[-2.80, -1.90]$	$[-2.97, -2.16]$
3	Semiconductor Lithography and Fabrication Processes	2.48	$[-0.85, -0.79]$	$[-0.66, -0.63]$	$[-0.85, -0.80]$	$[-0.66, -0.63]$
4	Integrated Circuit Design, Verification, and Testing	4.53	$[-3.14, -2.05]$	$[-3.19, -2.24]$	$[-3.14, -2.05]$	$[-3.19, -2.24]$
5	PCB and Electronic Hardware with Connectors	4.50	$[3.15, 3.55]$	$[2.88, 3.26]$	$[3.20, 3.55]$	$[2.88, 3.26]$
6	Video Recording, Encoding, and Media Processing	1.98	$[-0.88, -0.07]$	$[-0.62, 0.02]$	$[-0.88, -0.07]$	$[-0.62, 0.02]$
7	Semiconductor Packaging and Interconnect Bonding	2.57	$[-0.98, -0.92]$	$[-0.86, -0.81]$	$[-0.98, -0.92]$	$[-0.86, -0.81]$
8	Thin-Film Materials, Deposition and Chemistry	3.51	$[4.53, 4.70]$	$[4.27, 4.41]$	$[4.57, 4.70]$	$[4.27, 4.41]$
9	Wireless Base-Station and Mobile Communications	2.42	$[-1.20, -0.97]$	$[-0.99, -0.79]$	$[-1.20, -0.98]$	$[-0.99, -0.79]$
10	Clock Generation, Timing Circuits, and Signal Processing	4.06	$[-2.28, -1.67]$	$[-2.42, -1.89]$	$[-2.28, -1.67]$	$[-2.42, -1.89]$
11	Power-Supply and Power-Management Electronics	3.68	$[-2.36, -1.66]$	$[-2.22, -1.70]$	$[-2.36, -1.66]$	$[-2.22, -1.70]$
12	Networking Protocols and Data Infrastructure	2.47	$[-0.96, -0.70]$	$[-0.76, -0.55]$	$[-0.96, -0.72]$	$[-0.76, -0.56]$
13	Transistor Gate Structures and Semiconductor Device Physics	2.03	$[-0.69, -0.02]$	$[-0.50, 0.08]$	$[-0.69, -0.02]$	$[-0.50, 0.08]$
14	Imaging Sensors and Camera Pipelines	1.91	$[-0.53, 0.67]$	$[-0.24, 0.85]$	$[-0.53, 0.67]$	$[-0.24, 0.85]$
15	Processor Architecture, Caches, Instruction Execution	3.08	$[-1.69, -1.41]$	$[-1.72, -1.49]$	$[-1.69, -1.41]$	$[-1.72, -1.49]$
16	Display Panel Tech (LCD/OLED)	1.02	$[7.32, 7.82]$	$[7.02, 7.53]$	$[7.32, 7.82]$	$[7.02, 7.53]$
17	Rotational Electromechanics and Magnetic Actuation	4.39	$[3.38, 3.69]$	$[3.12, 3.39]$	$[3.48, 3.69]$	$[3.17, 3.39]$
18	Optics, Lasers, and Photonics	2.21	$[-0.81, -0.42]$	$[-0.51, -0.18]$	$[-0.81, -0.42]$	$[-0.51, -0.18]$
19	Analog and Differential Amplifier Circuits	3.57	$[-1.90, -1.32]$	$[-1.90, -1.43]$	$[-1.90, -1.32]$	$[-1.90, -1.43]$
20	Mobile UI and Content Services	3.45	$[-2.13, -1.64]$	$[-2.03, -1.64]$	$[-2.13, -1.64]$	$[-2.03, -1.64]$

Notes: Cells are the minimum and maximum over the 99 percent confidence set for $\theta = (\alpha, \beta, \kappa)$ under the specification of Table 9 that uses all three instruments and the largest subset sample, pairwise interactions among entered areas, and add and drop deviations of size two and three. Rival counts are held at their observed values. When a direction is reported, it holds for every vector in that set. The per-area cost in PCB and Electronic Hardware with Connectors, Thin-Film Materials, Deposition and Chemistry, Display Panel Tech (LCD/OLED), and Rotational Electromechanics and Magnetic Actuation is reduced by 20 or 50 percent. Fixed k re-ranks the observed portfolio, so a larger share in the subsidized areas is a smaller share in the others. With adds, I also occupy unentered areas that would raise the observable part of profit. Δ is the change in the area share relative to the observed full-sample share.

Table 26: Hardware cost subsidy, 2010s sample

Topic	Label	Observed (%)	Δ at 20% (fixed k)	Δ at 20% (with adds)	Δ at 50% (fixed k)	Δ at 50% (with adds)
1	Semiconductor Memory Architecture	1.78	$[-0.46, 1.48]$	$[0.08, 1.74]$	$[-0.46, 1.48]$	$[0.08, 1.74]$
2	Audio and Speech-Signal Processing	4.76	$[-2.60, -1.63]$	$[-2.86, -2.01]$	$[-2.60, -1.63]$	$[-2.86, -2.01]$
3	Semiconductor Lithography and Fabrication Processes	2.67	$[-0.74, -0.58]$	$[-0.47, -0.33]$	$[-0.74, -0.59]$	$[-0.47, -0.33]$
4	Integrated Circuit Design, Verification, and Testing	4.64	$[-2.70, -1.69]$	$[-2.86, -1.97]$	$[-2.70, -1.69]$	$[-2.86, -1.97]$
5	PCB and Electronic Hardware with Connectors	4.64	$[2.88, 3.20]$	$[2.45, 2.77]$	$[2.94, 3.20]$	$[2.45, 2.77]$
6	Video Recording, Encoding, and Media Processing	2.08	$[-0.96, 0.36]$	$[-0.42, 0.60]$	$[-0.96, 0.36]$	$[-0.42, 0.60]$
7	Semiconductor Packaging and Interconnect Bonding	3.23	$[-1.20, -1.01]$	$[-1.07, -0.89]$	$[-1.20, -1.01]$	$[-1.07, -0.89]$
8	Thin-Film Materials, Deposition and Chemistry	3.35	$[4.45, 4.63]$	$[4.02, 4.19]$	$[4.47, 4.63]$	$[4.02, 4.19]$
9	Wireless Base-Station and Mobile Communications	3.07	$[-1.49, -1.29]$	$[-1.21, -1.04]$	$[-1.49, -1.29]$	$[-1.23, -1.04]$
10	Clock Generation, Timing Circuits, and Signal Processing	4.20	$[-2.01, -1.57]$	$[-2.20, -1.85]$	$[-2.01, -1.57]$	$[-2.20, -1.85]$
11	Power-Supply and Power-Management Electronics	4.08	$[-2.26, -1.41]$	$[-2.10, -1.51]$	$[-2.26, -1.41]$	$[-2.10, -1.51]$
12	Networking Protocols and Data Infrastructure	2.91	$[-1.28, -1.12]$	$[-0.98, -0.86]$	$[-1.28, -1.17]$	$[-0.98, -0.88]$
13	Transistor Gate Structures and Semiconductor Device Physics	2.26	$[-0.64, 0.18]$	$[-0.37, 0.35]$	$[-0.64, 0.18]$	$[-0.37, 0.35]$
14	Imaging Sensors and Camera Pipelines	2.61	$[-0.92, -0.66]$	$[-0.58, -0.37]$	$[-0.92, -0.66]$	$[-0.58, -0.37]$
15	Processor Architecture, Caches, Instruction Execution	3.54	$[-1.73, -1.38]$	$[-1.76, -1.50]$	$[-1.73, -1.38]$	$[-1.77, -1.50]$
16	Display Panel Tech (LCD/OLED)	1.15	$[6.80, 7.10]$	$[6.35, 6.66]$	$[6.80, 7.10]$	$[6.35, 6.66]$
17	Rotational Electromechanics and Magnetic Actuation	4.29	$[3.55, 3.79]$	$[3.11, 3.36]$	$[3.59, 3.79]$	$[3.11, 3.36]$
18	Optics, Lasers, and Photonics	2.48	$[-0.70, -0.20]$	$[-0.25, 0.14]$	$[-0.70, -0.20]$	$[-0.25, 0.14]$
19	Analog and Differential Amplifier Circuits	3.78	$[-1.81, -1.34]$	$[-1.81, -1.48]$	$[-1.81, -1.34]$	$[-1.81, -1.48]$
20	Mobile UI and Content Services	3.93	$[-1.85, -1.22]$	$[-1.79, -1.27]$	$[-1.85, -1.22]$	$[-1.79, -1.27]$

Notes: Cells are the minimum and maximum over the 99 percent confidence set for $\theta = (\alpha, \beta, \kappa)$ under the specification of Table 9 that uses all three instruments and the largest subset sample, pairwise interactions among entered areas, and add and drop deviations of size two and three. Rival counts are held at their observed values. When a direction is reported, it holds for every vector in that set. The per-area cost in PCB and Electronic Hardware with Connectors, Thin-Film Materials, Deposition and Chemistry, Display Panel Tech (LCD/OLED), and Rotational Electromechanics and Magnetic Actuation is reduced by 20 or 50 percent. The sample is 2010s firm-years. Valuations, rival counts, and observed portfolios are those of the 2010s. Fixed k re-ranks the observed 2010s portfolio, so a larger share in the subsidized areas is a smaller share in the others. With adds, I also occupy unentered areas that would raise the observable part of profit. Δ is the change in the area share relative to the observed 2010s share.

Table 27: Semiconductor cost subsidy, 2010s sample

Topic	Label	Observed (%)	Δ at 20% (fixed k)	Δ at 20% (with adds)	Δ at 50% (fixed k)	Δ at 50% (with adds)
1	Semiconductor Memory Architecture	1.78	$[5.69, 6.36]$	$[5.23, 5.91]$	$[5.69, 6.36]$	$[5.23, 5.91]$
2	Audio and Speech-Signal Processing	4.76	$[-2.95, -2.37]$	$[-3.16, -2.69]$	$[-2.95, -2.37]$	$[-3.16, -2.69]$
3	Semiconductor Lithography and Fabrication Processes	2.67	$[4.49, 4.98]$	$[4.04, 4.53]$	$[4.49, 4.98]$	$[4.04, 4.53]$
4	Integrated Circuit Design, Verification, and Testing	4.64	$[1.51, 2.44]$	$[1.03, 2.00]$	$[1.54, 2.44]$	$[1.04, 2.00]$
5	PCB and Electronic Hardware with Connectors	4.64	$[-2.95, -2.23]$	$[-2.76, -2.29]$	$[-2.96, -2.23]$	$[-2.77, -2.30]$
6	Video Recording, Encoding, and Media Processing	2.08	$[-1.21, -0.54]$	$[-0.70, -0.29]$	$[-1.21, -0.54]$	$[-0.70, -0.29]$
7	Semiconductor Packaging and Interconnect Bonding	3.23	$[3.63, 4.02]$	$[3.22, 3.58]$	$[3.78, 4.02]$	$[3.28, 3.58]$
8	Thin-Film Materials, Deposition and Chemistry	3.35	$[-1.98, -1.80]$	$[-1.60, -1.43]$	$[-1.98, -1.95]$	$[-1.61, -1.56]$
9	Wireless Base-Station and Mobile Communications	3.07	$[-1.94, -1.86]$	$[-1.64, -1.55]$	$[-1.94, -1.86]$	$[-1.66, -1.55]$
10	Clock Generation, Timing Circuits, and Signal Processing	4.20	$[-2.48, -2.08]$	$[-2.54, -2.30]$	$[-2.48, -2.08]$	$[-2.54, -2.30]$
11	Power-Supply and Power-Management Electronics	4.08	$[-2.62, -2.27]$	$[-2.46, -2.28]$	$[-2.62, -2.27]$	$[-2.48, -2.28]$
12	Networking Protocols and Data Infrastructure	2.91	$[-1.67, -1.47]$	$[-1.38, -1.26]$	$[-1.67, -1.55]$	$[-1.38, -1.29]$
13	Transistor Gate Structures and Semiconductor Device Physics	2.26	$[5.34, 5.68]$	$[4.89, 5.23]$	$[5.34, 5.68]$	$[4.89, 5.23]$
14	Imaging Sensors and Camera Pipelines	2.61	$[-1.53, -1.31]$	$[-1.09, -0.97]$	$[-1.53, -1.31]$	$[-1.09, -0.97]$
15	Processor Architecture, Caches, Instruction Execution	3.54	$[2.86, 3.41]$	$[2.45, 2.97]$	$[3.06, 3.41]$	$[2.56, 2.97]$
16	Display Panel Tech (LCD/OLED)	1.15	$[-0.47, 1.16]$	$[0.25, 1.52]$	$[-0.47, 1.16]$	$[0.25, 1.51]$
17	Rotational Electromechanics and Magnetic Actuation	4.29	$[-2.66, -2.39]$	$[-2.35, -2.14]$	$[-2.66, -2.39]$	$[-2.35, -2.20]$
18	Optics, Lasers, and Photonics	2.48	$[-1.31, -1.03]$	$[-0.83, -0.62]$	$[-1.31, -1.03]$	$[-0.83, -0.62]$
19	Analog and Differential Amplifier Circuits	3.78	$[-2.20, -2.00]$	$[-2.09, -1.99]$	$[-2.20, -2.00]$	$[-2.09, -1.99]$
20	Mobile UI and Content Services	3.93	$[-2.34, -2.04]$	$[-2.30, -2.08]$	$[-2.34, -2.04]$	$[-2.31, -2.08]$

Notes: Cells are the minimum and maximum over the 99 percent confidence set for $\theta = (\alpha, \beta, \kappa)$ under the specification of Table 9 that uses all three instruments and the largest subset sample, pairwise interactions among entered areas, and add and drop deviations of size two and three. Rival counts are held at their observed values. When a direction is reported, it holds for every vector in that set. The per-area cost in Semiconductor Memory Architecture, Semiconductor Lithography and Fabrication Processes, Integrated Circuit Design, Verification, and Testing, Semiconductor Packaging and Interconnect Bonding, Transistor Gate Structures and Semiconductor Device Physics, and Processor Architecture, Caches, Instruction Execution is reduced by 20 or 50 percent. The sample is 2010s firm-years. Valuations, rival counts, and observed portfolios are those of the 2010s. Fixed k re-ranks the observed 2010s portfolio, so a larger share in the subsidized areas is a smaller share in the others. With adds, I also occupy unentered areas that would raise the observable part of profit. Δ is the change in the area share relative to the observed 2010s share.

Table 28: Change in area shares under a connectivity cost subsidy

Topic	Label	Observed (%)	Δ at 20% (fixed k)	Δ at 20% (with adds)	Δ at 50% (fixed k)	Δ at 50% (with adds)
1	Semiconductor Memory Architecture	1.87	$[-0.49, 0.55]$	$[-0.24, 0.64]$	$[-0.49, 0.55]$	$[-0.24, 0.64]$
2	Audio and Speech-Signal Processing	4.36	$[-2.72, -1.71]$	$[-2.85, -1.91]$	$[-2.72, -1.71]$	$[-2.85, -1.91]$
3	Semiconductor Lithography and Fabrication Processes	2.48	$[-0.86, -0.82]$	$[-0.76, -0.72]$	$[-0.86, -0.83]$	$[-0.77, -0.72]$
4	Integrated Circuit Design, Verification, and Testing	4.53	$[-3.07, -1.87]$	$[-3.14, -2.07]$	$[-3.07, -1.87]$	$[-3.14, -2.07]$
5	PCB and Electronic Hardware with Connectors	4.50	$[-2.78, -1.70]$	$[-2.67, -1.76]$	$[-2.78, -1.70]$	$[-2.67, -1.76]$
6	Video Recording, Encoding, and Media Processing	1.98	$[-0.55, 0.14]$	$[-0.37, 0.21]$	$[-0.55, 0.14]$	$[-0.37, 0.21]$
7	Semiconductor Packaging and Interconnect Bonding	2.57	$[-0.96, -0.87]$	$[-0.93, -0.83]$	$[-0.96, -0.88]$	$[-0.93, -0.83]$
8	Thin-Film Materials, Deposition and Chemistry	3.51	$[-1.94, -1.42]$	$[-1.76, -1.27]$	$[-1.94, -1.42]$	$[-1.76, -1.27]$
9	Wireless Base-Station and Mobile Communications	2.42	$[6.18, 6.31]$	$[5.89, 6.02]$	$[6.18, 6.31]$	$[5.89, 6.04]$
10	Clock Generation, Timing Circuits, and Signal Processing	4.06	$[-2.19, -1.40]$	$[-2.29, -1.59]$	$[-2.19, -1.40]$	$[-2.29, -1.59]$
11	Power-Supply and Power-Management Electronics	3.68	$[-2.18, -1.40]$	$[-2.17, -1.50]$	$[-2.18, -1.40]$	$[-2.17, -1.50]$
12	Networking Protocols and Data Infrastructure	2.47	$[6.18, 6.31]$	$[5.89, 6.03]$	$[6.19, 6.31]$	$[5.89, 6.06]$
13	Transistor Gate Structures and Semiconductor Device Physics	2.03	$[-0.61, 0.03]$	$[-0.47, 0.09]$	$[-0.61, 0.03]$	$[-0.47, 0.09]$
14	Imaging Sensors and Camera Pipelines	1.91	$[-0.38, 0.81]$	$[-0.15, 0.92]$	$[-0.38, 0.81]$	$[-0.15, 0.92]$
15	Processor Architecture, Caches, Instruction Execution	3.08	$[-1.52, -1.08]$	$[-1.55, -1.14]$	$[-1.52, -1.08]$	$[-1.55, -1.14]$
16	Display Panel Tech (LCD/OLED)	1.02	$[0.87, 4.06]$	$[1.35, 4.23]$	$[0.87, 4.06]$	$[1.35, 4.23]$
17	Rotational Electromechanics and Magnetic Actuation	4.39	$[-2.60, -1.79]$	$[-2.45, -1.76]$	$[-2.60, -1.79]$	$[-2.45, -1.76]$
18	Optics, Lasers, and Photonics	2.21	$[-0.78, -0.39]$	$[-0.52, -0.18]$	$[-0.78, -0.39]$	$[-0.52, -0.18]$
19	Analog and Differential Amplifier Circuits	3.57	$[-1.79, -1.18]$	$[-1.87, -1.34]$	$[-1.79, -1.18]$	$[-1.87, -1.34]$
20	Mobile UI and Content Services	3.45	$[4.62, 4.94]$	$[4.37, 4.65]$	$[4.69, 4.94]$	$[4.37, 4.65]$

Notes: Cells are the minimum and maximum over the 99 percent confidence set for $\theta = (\alpha, \beta, \kappa)$ under the specification of Table 9 that uses all three instruments and the largest subset sample, pairwise interactions among entered areas, and add and drop deviations of size two and three. Rival counts are held at their observed values. When a direction is reported, it holds for every vector in that set. The per-area cost in Wireless Base-Station and Mobile Communications, Networking Protocols and Data Infrastructure, and Mobile UI and Content Services is reduced by 20 or 50 percent. Fixed k re-ranks the observed portfolio, so a larger share in the subsidized areas is a smaller share in the others. With adds, I also occupy unentered areas that would raise the observable part of profit. Δ is the change in the area share relative to the observed full-sample share.